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Effects of the Pion String at Heavy Ion Collisions *

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Abstract We study the possible signals of the pion string associated with the QCD chiral phase transition in LHC Pb-Pb collision at energy $\sqrt{s}=5.5$ TeV. We follow the Kibble-Zurek mechanism to discuss the production and evolution of the pion string. We will show that if the QCD chiral phase transition really takes place in the LHC Pb-Pb collision process and the phase transition is in the second order, the pion string will be inevitably produced and subsequently decay. The main effect of this phenomenon is that there is a generation of a large number of pions in the final state produced by the decay of the pion string, and these pions are mostly distributed in a low momentum region with $p\sim 143$MeV; also there are lots of neutral pions distributed in a low momentum region with the mean momentum at $p\sim 21$MeV.

Key words π string, Kibble-Zurek mechanism, heavy ion collisions

Several years ago Ref.[1] discovered a type of classical solution, the pion string, in the linear sigma model of $SU(2)_L\times SU(2)_R\sim O(4)$ which had been studied widely in the literature for the description of the QCD chiral symmetry breaking. This pion string is very much like the spin vortex produced at the superfluid transition of the liquid ^{3}He[2]. Just like the Z string[3] in the standard electroweak model, the pion string is not topologically stable since any field configuration can be continuously deformed to the trivial vacuum in the QCD sigma model. With finite temperature plasma, however, Ref.[4] argued that the pion string can be stabilized. They propose that the interaction of the pion fields with the charged plasma generates a correction to the effective potential and this correction reduces the vacuum maniford S^3 of the zero temperature theory to a lower dimensional sub-maniford S^1, which makes the pion string stable. Moreover, it has been shown by numerical simulations that semilocal strings, which are also not topologically stable, can be produced at the phase transition[5]. In a similar way, pion strings are expected to be produced during the QCD phase transition in the early universe as well as in experiment of the heavy ion collisions. In Ref.[6] the effect of the pion string on the primordial magnetic field generation in the early universe has been considered and its cosmological significance is pointed out. In this paper we study a possible signal of the pion string in the heavy ion collision and show another kind of the significance of the pion string in laboratory experiments.

The Lagrangian of the $SU(2)_L\times SU(2)_R\sim O(4)$ linear sigma model is given by

$$\mathscr{L}_\Phi=\mathrm{Tr}[(\partial_\mu\Phi)^+(\partial^\mu\Phi)]-\lambda\left[\mathrm{Tr}(\Phi^+\Phi)-\frac{f_\pi^2}{2}\right]^2, \quad (1)$$

where $\Phi=\sigma\frac{\tau^0}{2}+i\boldsymbol{\pi}\cdot\frac{\boldsymbol{\tau}}{2}$, τ^0 is the unity matrix and $\boldsymbol{\tau}$ is the Pauli matrices with the normalization condition $\mathrm{Tr}(\tau^a\tau^b)=2\delta^{ab}$. During chiral symmetry breaking, the field σ takes on a non-vanishing vacuum expectation value, which breaks $SU(2)_L\times SU(2)_R$ down to $SU(2)_{L+R}$. This results in a massive sigma and three massless Goldstone bosons.

In order to investigate the formation and the evolution of the pion strings, we define

$$\phi=\frac{\sigma+i\pi^0}{\sqrt{2}},\ \pi^\pm=\frac{\pi^1\pm i\pi^2}{\sqrt{2}}. \quad (2)$$

The Lagrangian $\mathscr{L}_\Phi$ now can be rewritten as

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$$\mathscr{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) + (\partial_\mu \pi^+)(\partial^\mu \pi^-) - \lambda\left(\phi^*\phi + \pi^+\pi^- - \frac{f_\pi^2}{2}\right)^2. \quad (3)$$

For the static configuration, the energy functional corresponding to the Lagrangian above is given by

$$E = \int d^3x\left[\nabla\phi^* \nabla\phi + \nabla\pi^+ \nabla\pi^- + \lambda\left(\phi^*\phi + \pi^+\pi^- - \frac{f_\pi^2}{2}\right)^2\right]. \quad (4)$$

The pion string solution with a single winding number extremizing the energy functional in Eq. (4) is given by[1]

$$\phi = \frac{f_\pi}{\sqrt{2}}[1 - \exp(-\mu r)]\exp(i\theta) \quad (5)$$

and

$$\pi^\pm = 0, \quad (6)$$

where the coordinates r and θ are polar coordinates in x-y plane (the string is assumed to lie along the z axis), $\mu^2 = \lambda \frac{89}{144} f_\pi^2$. The energy per unit length of the string is

$$E = [0.75 + \log(\mu R)]\pi f_\pi^2, \quad (7)$$

where R is introduced as a cut off since for global symmetry the energy density of the string solution is logarithmically divergent. Generally R is given by the horizon size or the typical separation length between strings. In the following numerical calculation, we take $R = O(\mathrm{fm})$, for other parameters we have $\lambda = 9.877$, $f_\pi = 90\mathrm{MeV}$, $m_\pi = 140\mathrm{MeV}$ and $m_\sigma = 400\mathrm{MeV}$[7].

According to Pisarski and Wilczek[8] the chiral phase transition is expected to be of the second order for two massless flavors, it is customary then to apply the Kibble-Zurek mechanism[9,10] in order to study the formation and evolution of the pion string during Pb-Pb central collisions at the LHC with energy $\sqrt{s} = 5.5\mathrm{TeV}$. At the initial stage of the collision there exist manifestly partons with very large cross section for gluon scattering, so the gluons will reach equilibrium quickly with an initial temperature at about $T_i = 600\mathrm{MeV}$ corresponding to the time $t_i = 0.2\mathrm{fm}$[11]. If the entropy of the system is conserved throughout the expansion, using the Bjorken model we see that the thermal freeze out of the fireball occurs at $t_f = 25\mathrm{fm}$ when the temperature reaches $T_f = 120\mathrm{MeV}$. The calculation in Ref. [12] shows that the quark-gluon plasma can be formed over very large space-time volumes at the LHC Pb-Pb collisions. The hydrodynamic model predicts that the volume of such a plasma region evolves as[13—15],

$$V(t_f) = V(t_c)\frac{t_f}{t_c}, \quad (8)$$

where $V(t_f) = 2\times10^4\mathrm{fm}^3$, while evolution of the temperature is given by

$$T(t) = T_i\left(\frac{t_i}{t}\right)^{\frac{1}{3}}. \quad (9)$$

From Eqs. (8) and (9) we obtain that the time when the phase transition occurs at $T_c = 170\mathrm{MeV}$ is $t_c \simeq 8.793\mathrm{fm}$, and the volume at the freezing out is $V(t_f) = 2\times10^4\mathrm{fm}^3$.

When the LHC Pb-Pb collisions take place, a big fireball is formed in the central region of the collision with the initial temperature around $T(t_i) = 600\mathrm{MeV}$ at the time $t_i = 0.2\mathrm{fm}$. The fireball quickly reaches to the equilibrium state and it expands rapidly with the volume and temperature given in Eqs. (8) and (9). During the period when the temperature is higher than $T_c = 170\mathrm{MeV}$ (before $t_c \simeq 8.793\mathrm{fm}$), the fireball is in the QGP phase where the chiral symmetry is unbroken. When the temperature of the fireball decreases down to the critical point $T_c(t_c) = 170\mathrm{MeV}$ and its volume is about $V(t_c) \approx 7\times10^3\mathrm{fm}^3$, the fireball undergoes a rapid second order chiral phase transition. At this time the system is in an out of equilibrium dynamical state, and the phase with broken chiral symmetry starts to appear due to the fluctuations of the order parameter simultaneously and independently in many separate regions of the expanding fireball. Subsequently during the process with further cooling, these regions grow and merge with each other to realize the new phase with the broken symmetry all over the fireball. At the boundaries where causally disconnected different regions meet, the order parameter field does not necessarily match and a domain structure is formed. This is essentially similar to the process of the defect formation during the cosmological phase transitions in the early universe.

As described by the Kibble-Zurek mechanism, the transition speed can be given by the quench time τ_Q[9,10,16],

$$\tau_Q = \frac{T_c}{|dT/dt|_{t=t_c}} = 3t_c. \quad (10)$$

From the Ginzburg-Landau theory for the second order phase transition, the quench time τ_Q can be deduced by the order parameter relaxation time τ with a general form

$$\tau(T) = \tau_0\left(1 - \frac{T}{T_c}\right)^{-1}, \quad (11)$$

where $\tau_0 \sim \xi_0$ and ξ_0 is the zero temperature limiting value of the temperature dependent coherence length $\xi(T)$, which in this paper we take $\xi_0 = 1/m_\sigma \simeq 0.49\mathrm{fm}$. When the temperature T is close to T_c,

$$\xi(T) = \xi_0\left(1 - \frac{T}{T_c}\right)^{-\frac{1}{2}}. \quad (12)$$

As the temperature is below T_c the order parameter coherence spreads out with the velocity

$$c(T) \sim \frac{\xi}{\tau} = \frac{\xi_0}{\tau_0}\left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}. \qquad (13)$$

The pion strings are expected to be produced at the Zurek freeze-out time t_z when the causally disconnected regions have grown together and the coherence becomes established in the whole volume. At the Zurek freeze-out temperature $T(t_z) < T_c$, the causal horizon is given by

$$\xi_H(t_z) = \int_0^{t_z} c(T)\mathrm{d}t = \frac{\xi_0 \tau_Q}{\tau_0}\left(1 - \frac{T_z}{T_c}\right)^{\frac{3}{2}}. \qquad (14)$$

The causal horizon has to be equal to the coherence length $\xi(t_z)$, then from Eqs. (11), (12), (14) we obtain

$$t_z = t_c + \tau(t_z) = t_c + \sqrt{\tau_0 \tau_Q} \approx 12.4\text{fm}, \qquad (15)$$

and

$$\xi_z = \xi_0(\tau_Q/\tau_0)^{1/4} \simeq \tau_Q^{1/4}\tau_0^{3/4} \simeq 1.33\text{fm}. \qquad (16)$$

Just like the evolution of the cosmic string in the early universe[6,17,18], when the temperature of the fireball falls down from the Zurek temperature, $T_z = T_i\left(\frac{t_i}{t_z}\right)^{\frac{1}{3}} = 151.6\text{MeV}$ to T_f, the evolution of the string network would obey the following procedure. Initially, at the Zurek time t_z, the pion string has a typical curvature radius and separation of the correlation length ξ_z, which then increases rapidly and eventually approaches a scaling solution in which $\xi(t) \sim t^a$. In the case of the pion string in the heavy ion collision experiments, since the volume of fireball obeys the following law $V(t) \sim t$, we make the assumption that $a = \frac{1}{3}$ for our case from time t_z up to t_f. Therefore, the correlation length of the pion string at time t_f is

$$\xi_f = \xi(t_f) = \xi_z\left(\frac{t_f}{t_z}\right)^{\frac{1}{3}} \simeq 1.69\text{fm}. \qquad (17)$$

The pion string ceases to evolve at the time t_f and decays because of the fireball disappearance. Thus we have to find out the size and number of loops at the time t_f.

Note that in our situation, closed string loops have a dominant contribution to the total energy of the string. As the string evolution is still ruled out by the frictional force by the surrounding matter at the freezing out, the free motion of the string is not realized. In addition, the expansion of the system is so rapid that the initial Brownian string distribution will be conserved. Then the initial structure of the string network partially remains and the spatial trajectories of strings are very much complicated. Thus we take the initial pion string network as that of the Brownian one and regard that the distribution of these loops does not change with time, the size of the loops are conformally stretched during the expansion of fireball and a simple scaling can be realized. By using the scale invariance of Brownian string described by Vachaspati and Vilenkin[18], we get the distribution of number density of the pion string with the length between l and $l + \mathrm{d}l$ at the freeze out temperature, T_f,

$$\mathrm{d}n(l) = K\xi_f^{-\frac{3}{2}} l^{-\frac{5}{2}}\mathrm{d}l, \qquad (18)$$

where the parameter K is approximately in the range of $0.01 \sim 0.1$[2].

Integration of Eq. (21) over $\mathrm{d}l$ will result in the total number of the string loops. Note that the string width $r_0 \sim \frac{1}{\mu}$ gives a minimum length of the pion string at time t_f, $l_0 = 2\pi r_0 \simeq 5.6\text{fm}$ and the longest string is also constrained by the volume of the system. From Eq. (18) we have the total number of pion strings

$$N(0) = \frac{2KV(t_f)}{3\xi_f^{\frac{3}{2}} l_0^{\frac{3}{2}}} = 459\text{K}. \qquad (19)$$

Since K varies from 0.01 to 0.1 the total number of pion strings varies between $N(0) \simeq 4$ and $N(0) \simeq 45$.

In the immediate aftermath of the phase transition, when the temperature is still close to T_c, the string tension remains small and motion of the strings is heavily damped by the frictional effects of the surrounding high-density medium. The mechanism of Nagasawa and Brandenberger implies that pion strings might effectively be stable in this high-density medium until the thermal freeze out time t_f. After that, the string tension approaches its zero-temperature value and the motion of the strings is effectively decoupled from the surrounding medium, there will not be corrections to the effective potential from the thermal bath and pion strings undergo the second phase transition. Even though pion strings undergo the second phase transition under the temperature T_f, pion strings will not decay immediately and can still survive sometime below the freeze out time t_f, since the pion strings undergo a core phase transition and lose their central structure where the field strength equals to zero but still preserve the winding number of neutral components[19]. For simplicity we assume that pion strings can survive after the decoupling time and all pions which are eventually emitted from pion strings will be completely incoherent with the rest of pions.

To estimate the numbers of the particle produced we notice that for the ansatz Eqs. (5) and (6), the sigma field in

Eq.(4) contributes about 50% of the total energy of the string. By energy conservation half of the string energy should convert into that carried by the sigma particle. The remaining 50% of the string energy will go to the neutral pions. For global string such as the axion string[20] one expects the mesons produced from the decay of the pion strings with length l have a typical momentum $p \sim 1/l$. Using Eq.(18), we obtain that the total number of sigma particle N_σ emitted from pion strings within a fireball is about 100($K=0.1$), 43 ($K=0.05$), 22($K=0.03$) and 5($K=0.01$), respectively. And the total number of neutral pions N_{π^0} is about 332 ($K=0.1$), 148($K=0.05$), 80($K=0.03$) and 19($K=0.01$). As mentioned above it is expected that the eventually resultant pion spectrum will have a nonthermal enhancement at low momentum region because all produced pions from pion strings are distributed at low momentum.

It can be seen that most of the sigma particles and the neutral pions have a relatively low momentum and these particles are nonthermal particles. The momentum distribution of the sigma from the pion string decay is given by

$$\frac{dN_\sigma(p)}{dp} = \frac{KV(t_f)\sqrt{p}}{D_1 \xi_f^{\frac{3}{2}}}, \tag{20}$$

where the normalization factor $D_1 = 0.476$. The momentum distribution of the neutral pions is given by

$$\frac{dN_{\pi^0}(p)}{dp} = \frac{KV(t_f)\sqrt{p}}{D_2 \xi_f^{\frac{3}{2}}}, \tag{21}$$

where the normalization factor $D_2 = 0.142$. The averaged momentum of sigma and neutral pion is $\langle p \rangle \simeq 21.1$MeV. The neutral pions and sigma paricles are dominant in the low momentum region, and the momentum distribution of the pions produced at the decay of the pion string can be taken as a distinctive signal of the formation of the pion string in heavy ion collisions.

The sigma particles from pion strings will decay equally into neutral and charged pion mesons. Since the momentum of the sigma can be approximately neglected, for $m_\sigma =$ 400MeV these pions from the sigma decay will have momentum around 143MeV. Numerically there are about 10 – 200 nonthermal pions mostly distributed at narrow region $p \sim$ [125.7, 161] MeV. This pion enhancement in a small window around the nonthermal momentum $p_0 \simeq 143$MeV will result in a peak in the pion spectra. Here, for simplicity, the large decay width of the sigma is ignored. We will take into account the effect of the broad decay width of the sigma particle and it will be published elsewhere.

Since the string configuration violates the isospin symmetry the direct production of the pions from the string decay is only for the neutral pion, not the charged pion. In Fig.1 we plot these neutral pion distribution together with the thermal pins calculated in Ref.[15]. Numerically there are about $N_{\pi^0} \sim 20 - 300$ nonthermal pions distributed in the low momentum region with $\langle p \rangle \sim 21$MeV and these nonthermal neutral pions will result in another peak in the pion spectra.

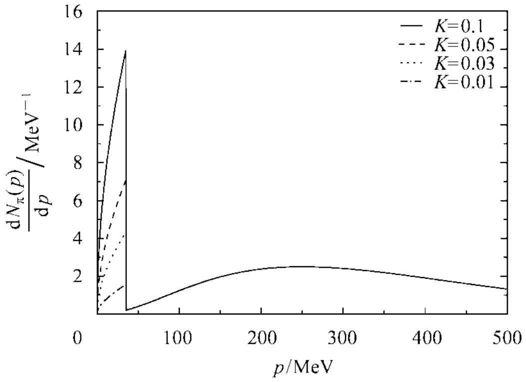

Fig.1. The momentum spectra of the nonthermal neutral pions (π^0's) emitted from the pion strings together with the thermal pions. The nonthermal pions are mainly distributed around $p \sim 0 - 35$MeV.

In summary, we have investigated the effects of the pion string existence in heavy ion collisions. The formation of the pion string and its decay are considered in LHC Pb-Pb collision at energy $\sqrt{s} = 5.5$ TeV. It is pointed out that if the QCD $SU(2)$ chiral phase transition really takes place in the LHC Pb-Pb collision process and the phase transition is of the second order, then the pion string will be formed by the Kibble-Zurek mechanism. The main consequence of the pion string formation is that there is a large number of pions ($N_\pi \sim 10 - 200$) produced at the final state of the fireball and these pions are mainly distributed in the low momentum region with $p \sim 143$MeV; moreover, there are lots of neutral pions ($N_{\pi^0} \sim 20 - 300$) distributed in the low momentum region with $\langle p \rangle \sim 21$MeV. These predictions can be easily distinguished from other models of heavy ion collision processes and can be clearly observed in the actual experiments[15,21]. It can give a confirmation that the QCD chiral phase transition is realized in high energy heavy ion collisions.

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π弦在重离子碰撞实验中的效应 *

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摘要 在能量为$\sqrt{s}=5.5$ TeV的LHC Pb-Pb重离子碰撞实验中,研究了和手征相变相关的π弦的可观测效应. 利用Kibble-Zurek机制讨论了π弦的产生和演化. 在LHC Pb-Pb重离子碰撞实验中,如果手征相变发生并且是二级相变,那么π弦将会产生然后衰变. π弦的主要效应是:π弦衰变成大量的末态π粒子,这些大量的π粒子,主要分布在动量为143MeV的低动量空间;同时π弦的衰变还伴随着大量中性π粒子,这些中性的π粒子主要分布在动量为21MeV的低动量空间.

关键词 π弦 Kibble-Zurek机制 重离子碰撞

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