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Pion Interferometry for Spherical Quark-Gluon Plasma Evolution Sources *

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Abstract We examine the two-pion interferometry for the expanding sources of spherical quark-gluon plasma evolution. The quark-gluon plasma evolution is described by relativistic hydrodynamics with the equation of state of entropy density. The two-pion Hanbury-Brown-Twiss (HBT) correlation functions are calculated using quantum probability amplitudes in a path-integral formalism. We find the spatial parameter extracted by the two-pion interferometry is sensitive to the phase-space distribution of the pion-emitting source. The expanding velocity of the source leads to a smaller HBT radius and changes the relationship between the HBT radius and the freeze-out temperature.

Key words pion interferometry, quark-gluon plasma, evolution source

Two-pion Hanbury-Brown-Twiss (HBT) interferometry is a useful tool to measure the size of the particle-emitting source in high energy heavy ion collisions[1]. At the energies of RHIC, or even SPS and AGS, a new phase of nuclear matter, the quark-gluon plasma (QGP), is expected to be formed[2]. The highest priority in ultrarelativistic heavy ion experiments is the detection of the new phase, QGP, and the study of its properties. In this paper, we examine the two-pion interferometry for the expanding sources which come from spherical quark-gluon plasma evolution. We follow Rischke and Gyulassy[3] and use the equation of state of entropy density suggested by QCD lattice data[4,5] to describe the quark-gluon plasma. Once the equation of state and the initial condition are known, the solution of the expansion and hadronization processes can be obtained by relativistic hydrodynamics without complicated microscopic details[3]. The two-pion correlation function, then, can be calculated by using quantum probability amplitudes in a path-integral formalism, after knowing the dynamical solution[6—8].

At zero net baryon density, the entropy density as a function of temperature can be expressed as[3—5]

$$\frac{s}{s_c}(T)=\left[\frac{T}{T_c}\right]^3\left(1+\frac{d_Q-d_H}{d_Q+d_H}\tanh\left[\frac{T-T_c}{\Delta T}\right]\right), \quad (1)$$

where d_Q and d_H are the degrees of freedom in the quark-gluon plasma phase and the hadronic phase, $T_c\approx 160\text{MeV}$ is the transition temperature, $s_c=\text{const.}\times\frac{1}{2}(d_Q+d_H)T_c^3$ is the entropy density at T_c, and ΔT (between 0 and $0.1T_c$) is the width of the transition[3]. From Eq. (1) one can get the pressure p, energy density ε, and the velocity of sound of the system with the following equations as in Ref. [3],

$$p=\int_0^T \mathrm{d}T' s(T'),\ \varepsilon=Ts-p,\ c_S^2=\frac{\mathrm{d}p}{\mathrm{d}\varepsilon}. \quad (2)$$

In this paper, we take $d_Q=37$, $d_H=3$, $T_c=160\text{MeV}$ as in Ref. [3], and consider the three kinds of systems of $\Delta T=0$ (an exact first-oder transition), $\Delta T=0.1\,T_c$, and the ideal pion gas with the equation of state $p=\varepsilon/3$.

The energy momentum tensor of a thermalized fluid cell in the center-of-mass frame of the source is[3,9,10]

$$T^{\mu\nu}(x)=[\varepsilon(x)+p(x)]u^\mu(x)u^\nu(x)-p(x)g^{\mu\nu}, \quad (3)$$

where x is the space-time coordinate, $u^\mu=\gamma(1,\boldsymbol{v})$ is the 4-velocity of the cell, and $g^{\mu\nu}$ is the metric tensor. From the local conservation of energy and momentum, one can get the

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equations for spherical geometry as[3]

$$\partial_t E + \partial_r[(E+p)v] = -F, \tag{4}$$

$$\partial_t M + \partial_r(Mv+p) = -G, \tag{5}$$

where $E \equiv T^{00}$, $M \equiv T^{0r}$,

$$F = \frac{2v}{r}(E+p),\ G = \frac{2v}{r}M. \tag{6}$$

We assume the initial conditions as[3]

$$\varepsilon(0,r) = \begin{cases} \varepsilon_0, & r < r_0, \\ 0, & r > r_0, \end{cases} \quad v(0,r) = \begin{cases} 0, & r < r_0, \\ 1, & r > r_0, \end{cases} \tag{7}$$

where ε_0 is the initial energy density, and r_0 is the initial radius of the system. In our calculations, we take $\varepsilon_0 = 1.875\, T_c s_c$[3], which is close to the energy density $\varepsilon_Q = \frac{1}{2}(4d_Q/d_H - 1)T_c s_c/(d_Q/d_H + 1) = 1.8125\, T_c s_c$ at the phase boundary between mixed phase and QGP for $\Delta T = 0$, and $r_0 = 6.0$ fm. Using the HLLE scheme[3] and with the relation of $p = p(\varepsilon)$ obtained from Eqs. (1) and (2), one can get the solutions of the hydrodynamical equations for $F = G = 0$, then, obtain the solutions for Eqs. (4) and (5) by using the Sod's operator splitting method[3,11]. The grid spacing for the HLLE scheme is taken as $\Delta x = 0.01 r_0$, and the time step width for the HLLE scheme and Sod's method corrector step is $\Delta t = 0.99\Delta x$[3]. Figs. 1 (1), (2), and (3) show the temperature profiles for the evolution systems of $\Delta T = 0$, $\Delta T = 0.1\ T_c$, and the ideal pion gas. Figs. 1 (1′), (2′), and (3′) show the velocity profiles, and (1″), (2″), and (3″) show the isotherms for the three systems, respectively.

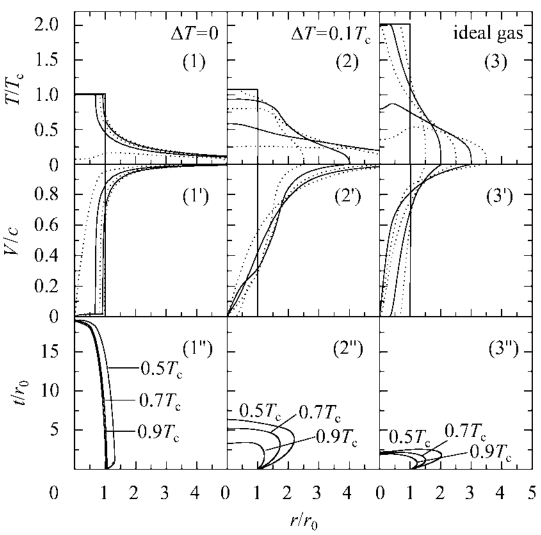

Fig. 1. (1,1′),(2,2′), and (3,3′) are the temperature and velocity profiles for the systems of $\Delta T = 0$ ($t_n = 4n\lambda r_0$), $\Delta T = 0.1\,T_c$ ($t_n = 1.5n\lambda r_0$), and the ideal pion gas ($t_n = 0.5n\lambda r_0$), respectively, where $n = 0,1,2,3,4,5$, $\lambda = 0.99$. (1″), (2″), and (3″) show the isotherms for the three systems.

The two-particle Bose-Einstein correlation function is defined as the ratio of the two-particle momentum distribution $P(k_1, k_2)$ to the product of the single-particle momentum distribution $P(k_1)\ P(k_2)$. Using quantum probability amplitudes in a path-integral formalism, $P(k_i)$ ($i = 1,2$) and $P(k_1, k_2)$ for an expanding source can be expressed as[6—8]

$$P(k) = \int d^4x\, e^{-2\mathrm{Im}\phi_s(x)} \rho(x) A^2(\kappa(x), x), \tag{8}$$

$$P(k_1,k_2) = \int d^4x_1 d^4x_2 e^{-2\mathrm{Im}\phi_s(x_1)} e^{-2\mathrm{Im}\phi_s(x_2)} \times \rho(x_1)\rho(x_2) \left|\Phi(\kappa_1\kappa_2 : x_1x_2 \to x_{d1}x_{d2})\right|^2, \tag{9}$$

where $e^{-2\mathrm{Im}\phi_s(x)}$ is the absorption factor due to multiple scattering[6—8], $\rho(x)$ is the pion-source density, $A(\kappa(x), x)$ is the magnitude of the amplitude for producing a pion with momentum k at x, and

$$\begin{aligned} &\Phi(\kappa_1\kappa_2 : x_1x_2 \to x_{d1}x_{d2} = \\ &\frac{1}{\sqrt{2}}\Big\{A(\kappa_1(x_1), x_1)A(\kappa_2(x_2)x_2) \times \\ &\exp\Big[-i\int_{x_1}^{x_{f1}} \kappa_1(x')\cdot dx' - ik_1\cdot(x_{d1} - x_{f1})\Big] \times \\ &\exp\Big[-i\int_{x_2}^{x_{f2}} \kappa_2(x')\cdot dx' - ik_2\cdot(x_{d2} - x_{f2})\Big] + \\ &A(\kappa_1(x_2), x_2)A(\kappa_2(x_1), x_1) \times \\ &\exp\Big[-i\int_{x_2}^{x'_{f2}} \kappa_1(x')\cdot dx' - ik_1\cdot(x_{d1} - x'_{f2})\Big] \times \\ &\exp\Big[-i\int_{x_1}^{x'_{f1}} \kappa_2(x')\cdot dx' - ik_2\cdot(x_{d2} - x'_{f1})\Big], \end{aligned} \tag{10}$$

is the wave function for two-pions produced at x_1 and x_2 with momenta $\kappa_1(x_1)$ and $\kappa_2(x_2)$, and detected at x_{d1} or x_{d2} with momenta k_1 and k_2, respectively. In Eq. (10), x_{f1} and x_{f2} are the freeze-out points corresponding to the pions 1 and 2 produced at x_1 and x_2 and detected at x_{d1} and x_{d2}, respectively. x'_{f1} and x'_{f2} are the freeze-out points corresponding to the pions 1 and 2 produced at x_1 and x_2 and detected at x_{d2} and x_{d1}, respectively. As we consider mainly two pions whose momenta are nearly parallel, we approximately have $x'_{f1} = x_{f1}$ and $x'_{f2} = x_{f2}$[6—8]. From Eqs. (8)—(10) the correlation function $C(k_1, k_2) = P(k_1, k_2)/P(k_1)P(k_2)$ can be written as[6—8]

$$C(k_1,k_2) = 1 + \left|\int d^4x\, e^{i(k_1-k_2)\cdot x + i\phi_c(x,k_1k_2)} \rho_{\mathrm{eff}}(x; k_1k_2)\right|^2, \tag{11}$$

where $\rho_{\rm eff}$ is the effective density

$$\rho_{\rm eff}(x;k_1k_2)=\frac{{\rm e}^{-2{\rm Im}\phi_{\rm s}(x)}\sqrt{f_{\rm init}(\kappa_1,x)f_{\rm init}(\kappa_2,x)}}{\sqrt{P(k_1)P(k_2)}},\qquad(12)$$

$$f_{\rm init}(\kappa,x)=\rho(x)A^2(\kappa(x),x),\qquad(13)$$

is the phase-space distribution of the pion-emitting source, which is proportional to the Bose-Einstein distribution in the local frame, and

$$\phi_{\rm c}(x,k_1k_2)=-\int_x^{x_{\rm f}}\{[\kappa_1(x')-\kappa_2(x')]-[k_1-k_2]\}\cdot{\rm d}x'.$$

From Eq.(12) it can be seen that the effective density is related to the phase-space distribution of the pion production source, modified by an absorption factor arising from multiple scattering. The two extreme cases of the absorption of multiple scattering are the pions without absorption after production and with a strong absorption which leads to a freeze-out emission[8]. In this paper we calculate the correlation functions for the three kinds of evolution sources, $\Delta T=0$, $\Delta T=0.1\,T_{\rm c}$, and ideal pion gas, and consider only the two extreme cases of the absorption of multiple scattering. We use the relative momentum and energy of the two pions, $q=|\boldsymbol{k}_1-\boldsymbol{k}_2|$ and $q_0=|E_1(\boldsymbol{k}_1)-E_2(\boldsymbol{k}_2)|$, as variables, the two-dimension correlation function $C(q,q_0)$ can be constructed from $P(k_1,k_2)$ and $P(k_1)P(k_2)$ by summing over $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ for each (q,q_0) bin. As the differences of the time scales of the three kinds of sources are large, we take small q_0 limitations when we study the spatial sizes of the three kinds of sources in order to reduce the effect of lifetime. We further obtain the one-dimension correlation function $C(q)$ by integrating $C(q,q_0)$ over q_0 within the limitations, which is 10MeV for the source of $\Delta T=0$ and 20MeV for the other sources.

Figs.2(1, 2, 3) and (1′, 2′, 3′) show the correlation function $C(q)$ for the three kinds of sources for the freeze-out temperature $T_{\rm f}=0.7T_{\rm c}$ and $0.9T_{\rm c}$, respectively. The symbols of bullets and circles are for the two cases of without absorption and with the freeze-out emission. The solid and dashed lines are the corresponding fitted curves with the parametrized correlation function as

$$C(q)=1+\lambda{\rm e}^{-q^2R^2}.\qquad(14)$$

Fig.3 gives the fitted results R and λ, where the symbols of bullets and circles are for the two cases of without absorption and with the freeze-out emission, respectively. It can be seen that the HBT radius for the freeze-out emission case is substantially greater than that for the corresponding case without

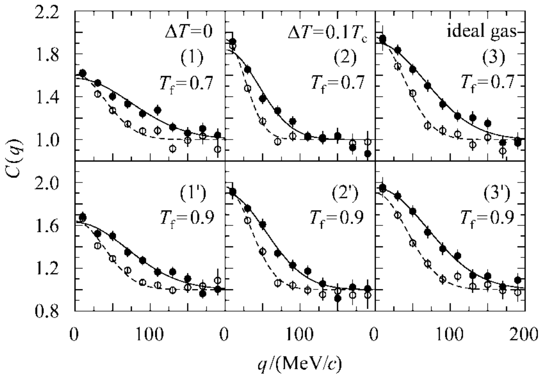

Fig.2. The two-pion correlation functions for the sources of $\Delta T=0$ (1 and 1′), $\Delta T=0.1\,T_{\rm c}$(2 and 2′), and ideal pion gas (3 and 3′) for the freeze-out temperatures $T_{\rm f}=0.7T_{\rm c}$ and $0.9T_{\rm c}$. The symbols of bullets and circles are for the cases of without absorption and with freeze-out emission, respectively.

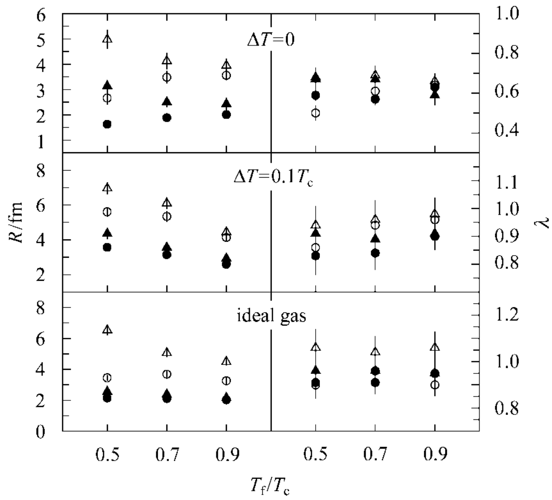

Fig.3. Two-pion HBT results for the sources of $\Delta T=0$, $\Delta T=0.1\,T_{\rm c}$, and ideal pion gas. The symbols of bullets and circles are for the cases of without absorption and with the freeze-out emission. The symbols of solid triangles and open triangles are the results for the cases of without absorption and with the freeze-out emission, corresponding to the "static" sources.

absorption. The HBT radius is sensitive to the phase-space distributions of the three kinds of sources. However, the HBT radius does not monotonously increase with decreasing the freeze-out temperature (refer to Figs. 1 (1″), (2″), and (3″)), because the average expanding velocities of the sources are different for different freeze-out temperatures. In order to investigate the effect of expanding velocity on HBT radius, we examine the two-pion interferometry for the corresponding "static" sources, which have the same density distributions as the expanding sources but the expanding velocities of the sources are forced to be zero. In Fig.3, the symbols of solid triangles and open triangles are the results for the "static" sources, for the cases of without absorption and with the freeze-out emission, respectively. It can be seen that the HBT radius of the "static" source increases monotonously with decreasing the freeze-out temperature both for the cases of without absorption and with the freeze-out emission. The expanding velocity of the source leads to a smaller HBT radius than that for the corresponding "static" source.

In summary, we examine the two-pion interferometry for the hydrodynamical evolution sources of spherical quark-gluon plasma, using quantum probability amplitudes in a path-integral formalism. The HBT radius is sensitive to the phase-space distribution of the pion-emitting source. The expanding velocity of the source leads to a smaller HBT radius. The different expanding velocities of the sources for different freeze-out temperatures lead to a change of the relationship between the HBT radius and the freeze-out temperature. We only considered two extreme cases of the absorption of multiple scattering in this paper. A more detailed investigation taking into consideration the effect of multiple scattering properly will be of great interest and possibly pave a way to understand the HBT puzzle in RHIC physics.

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球形夸克-胶子等离子体演化源的π干涉学分析*

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摘要 本文对球形演化的夸克-胶子等离子体膨胀源进行了2π干涉学分析. 夸克-胶子等离子体的演化由相对论流体力学和熵密度的物态方程描述, 而2π Hanbury-Brown-Twiss (HBT) 关联函数由量子几率振幅的路径积分公式计算. 研究结果表明, 由2π干涉学得到的源的空间参量敏感地依赖于π介子发射源的相空间分布, 源的膨胀速度导致HBT半径变小, 并会改变HBT半径与冻结温度之间的关系.

关键词 π干涉学 夸克-胶子等离子体 演化源

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