A Non-singularity form Factor of Baryon Vertex for Crossing Channel Study*

MA Wei-Xing[1] LIANG Wei-Hong[1,2] L.C.Liu[3]

1 (Institute of High Energy Physics, CAS, Beijing 100039, China)

2 (Department of Physics, Guangxi Normal University, Guilin 541004, China)

3 (Theoretical Division, Los Alamos National Laboratory, Los Alamos N M. 87545, USA)

Abstract A New relativistic and singularity free baryon form factor for crossing channel study is proposed. This new form factor satisfies the crossing symmetry law of strongly interacting particle scattering amplitudes transfered from *t*-channel to *s*-channel or the inverse.

Key words form factor, singularity, crossing symmetry

It is well known that baryon is not a point-like, structureless, fundamental particle but a sizeable object in space. A baryon consists of quarks and gluons with a volume occupied in space. Accordingly, when treating an interacting physical process involved a baryon a form factor described the internal quark-gluon structure of baryon must be used, which has been shown to be crucial in improving dramatically the theoretical description of experimental observables.

In the study of diffractive processes, the scattering amplitude of strongly interacting particle must satisfy the crossing symmetry required by **S**-matrix[1], since the **S**-matrix must be an analytic function and analytically continued from one channel to other channel. For example, when studying the asymptotic behavior of scattering amplitude for strongly interacting processes, the *t*-channel (a+ $\bar{c}$ → $\bar{b}$ +d) amplitude $A^t_{a+\bar{c}\to\bar{b}+d}(s, t)$ must be transfered into the amplitude of the *s*-channel (a+b →c+d) $A^s_{a+b\to c+d}(s, t)$, and the two amplitudes are required to satisfy the crossing symmetry relation[1]

$$A^{(t)}_{a+\bar{c}\to\bar{b}+d}(s_t,t_t,u_t)=A^{(s)}_{a+b\to c+d}(t,s,u) \quad , \tag{1}$$

where *s*, *t* and *u* are the mandelstam variables in *s*-channel and defined in the following way

$$s=(p_a+p_b)^2\,,\quad t=(p_a-p_c)^2\,,\quad u=(p_a-p_d)^2\,, \tag{2}$$

for the process a+b→c+d as shown in Fig.1.

* Supported by National Nature Science Foundation of China (19975053, 19835010, 10075081)

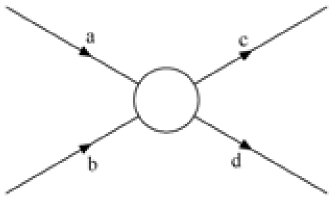

Fig.1. Schematic representation of two-body strong interaction in *s*-channel. The shady circle denotes strong interaction. The solid line represents strongly interacting particle a, b in the initial state and c, d in the final state.

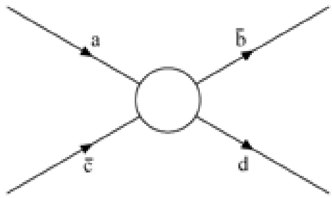

Fig.2. Schematic representation of two-body strong interaction in *t*-channel. The $\bar{b}$, $\bar{c}$ denote the corresponding anti-particles of the particles b and c, respectively.

The corresponding *t*-channel process $a+\bar{c}\rightarrow\bar{b}+d$ is shown in Fig.2, where the mandelstam variables s_t, t_t and u_t are defined in the same way as that in Eq. (2).

Unfortunately, all of commonly used form factors such as dipole form factor[2]

$$F_1(t)=\frac{4M_N^2-2.80t}{4M_N^2-t}\frac{1}{\left(1-\frac{t}{0.72}\right)^2}\quad , \tag{3}$$

used in diffractive processes, and exponential form factor[3]

$$F_{\exp}(t)= \exp(t/\lambda^2) \tag{4}$$

or the multipole form factor[4]

$$F_{\text{multi}}(t)=\left[\frac{\lambda^2}{\lambda^2-t}\right]^n, \quad n=1,2,\cdots \tag{5}$$

are unsuitable for analysing involving channel crossing. For example, the exponential form factor $F_{\exp}(t)$ in Eq. (4) is analytic in the *s*-channel where $t \le 0$, $s > 0$ but diverges in the *t*-channel where $t > 0$, $s \le 0$. The multipole form factor $F_{\text{multi}}(t)$ in Eq. (5) has no pole on the real axis of t in the *s*-channel where $t \le 0$ but will have it in the *t*-channel where $t > 0$. Conversely, if we use exp $(-t / \lambda_2)$ or $[\lambda^2/(\lambda^2+t)]^n$, then the situation will be reversed.

Needless to say, the dipole form factor $F_1(t)$ in Eq. (3) has many poles, such as $t = 4M^2$ and $t = 0.72\text{GeV}^2$. When $t=\frac{4M^2}{2.80}$, $F_1(t)=0$. Obviously, the dipole form factor $F_1(t)$ does not satisfy the crossing symmetry and cannot be used in any crossing channel calculation.

To overcome this difficulty, we need to propose in this note a new relativistic and singularity free form factor, which must satisfy the following physical requirements:

(1) It should be an analytic function of t so that it can be continued analytically from *s*-channel to *t*-channel, and vice verse.

(2) Since in the *t*-channel $t > 0$ and in the *s*-channel $t \le 0$, itshould behave as the following,

$$F_L^2(t)\xrightarrow{t\to\pm\infty}0 \quad .$$

(3) It should neither diverge with $t \to \pm\infty$, nor have a pole in the t, and it must be normalized as

$$F_{\mathrm{L}}^2\left(t = M_{\mathrm{res}}^2\right) = 1 \quad .$$

Therefore, the form factor may have the form as

$$F_{\mathrm{L}}^2(t) = \left(\frac{t/4 - M_{\mathrm{N}}^2}{q_{\mathrm{r}}^2}\right)^{\mathrm{L}} \cdot \left(\frac{\mathrm{e}^{t_{\mathrm{r}}/\lambda_t^2}}{R(-x_t) + \mathrm{e}^{t/\lambda_t^2}}\right)^2 \cdot \left(\frac{1 + \mathrm{e}^{-t_{\mathrm{r}}/\lambda_s^2}}{R(x_s) + \mathrm{e}^{-t/\lambda_s^2}}\right)^2 \quad , \tag{6}$$

where L is the orbital angular momentum of the relative motion of the particle. $(t/4 - M_{\mathrm{N}}^2)^{\mathrm{L}} \equiv f_{\mathrm{L}} = q^{2\mathrm{L}}$ reflects the q^{L}-dependence of the $F_{\mathrm{L}}(t)$. $t_{\mathrm{r}} = M_{\xi}^2$ with M_{ξ} being the mass of exchanged particle in t-channel. $\boldsymbol{q}$ and $\boldsymbol{q}_{\mathrm{r}}$ are , respectively, the off- and on-shell relative momentum in the c.m. system. The F_{L} has, therefore, the correct threshold q^{L}-dependence.

In Eq. (6), $x_s(t) \equiv (t - 2M_{\mathrm{N}}^2)/\lambda_{s(t)}^2$. The function $R(x)$ is analytic and is defined by

$$R(x) = \frac{1}{2}\left(1 + \tanh(ax)\right) = \frac{\mathrm{e}^{ax}}{\mathrm{e}^{ax} + \mathrm{e}^{-ax}} \quad . \tag{7}$$

It rapidly changes from 0 to 1 when x changes from negative value ($x < 0$) to positive one ($x > 0$), with a being controlling transition speed at $x=0$. The x-dependence of $R(x)$ with $a = 10$ is given in Fig.3.

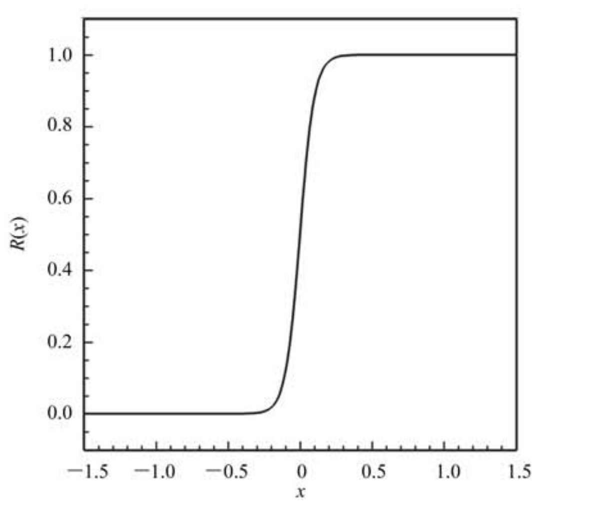

Fig. 3.　x-dependence of $R(x)$.

With $a > 10$, $R(x)$ is very close to a step function but does not have the discontinuity of the latter. Consequently, the form factor is a continuous function of t.

In the s-channel ($t \le 0$, $s > 0$)

$$F_{\mathrm{L}}^2(t) \propto f_{\mathrm{L}}\left[1 + \exp(t/\lambda_t^2)\right]^{-2} \cdot \left[\exp(-t/\lambda_s^2)\right]^{-2}) \tag{8}$$

$$\approx t^{\mathrm{L}} \exp(2t/\lambda_s^2) \quad , \tag{9}$$

which goes to 0 as $t \to -\infty$. Hence, λ_s controls the form factor. Since $F_L(t)$ does not have the physical-channel energy s as an explicit variable, it does not diverge with s.

In the t-channel ($t > 0$, $s \le 0$), when t reaches the t-channel physical domain $t > 4M_N^2$, we have

$$F_L^2(t) \propto f_L \left[\exp\left(t/\lambda_t^2\right)\right]^{-2} \cdot \left[1+\exp\left(-t/\lambda_s^2\right)\right]^{-2}$$
$$\approx t^L \exp\left(-2t/\lambda_t^2\right). \tag{10}$$

Therefore, when $t \to +\infty$, $F_L(t) \to 0$, exhibiting the correct energy behaviour in the t-channel.Hence in the t-channel the λ_t controls the form factor.

The above well-behaved t-dependence of $F_L(t)$ makes the form factor in Eq. (6) very good for continuing the scattering amplitudes between the direct and crossed channels. Notice that the form factor $F_L(t)$ in Eq. (6) proposed by us has no singularity in t. Particularly, when one studies on mass shell behavior, $t = M_{res}^2$, one has $F_L(t = M_{res}^2) = 1$. This is the normalization of form factor used to date.

The t-dependence of $F_L^2(t)$ is shown in Fig.4. In order to comparing with dipole form factor $F_1(t)$, Fig.5 shows the behaviour of $F_1(t)$ and its comparison with $F_L^2(t)$.

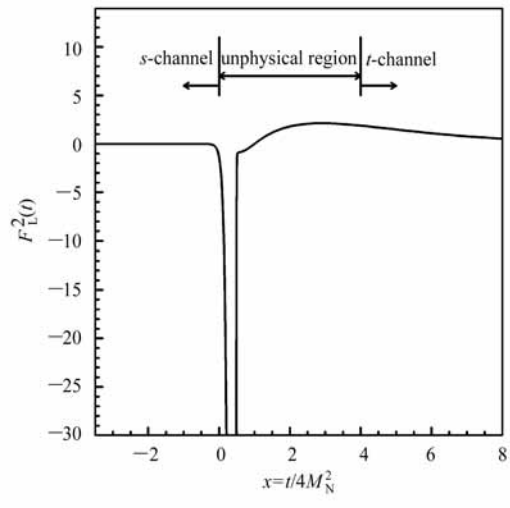

Fig. 4. Behaviour of $F_L^2(t)$ as t variation.

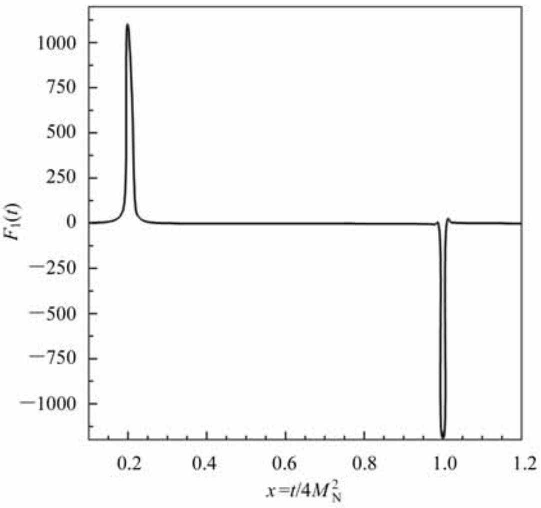

Fig. 5. Behaviour of $F_1(t)$ as t variation.

With the new form factor $F_L^2(t)$ defined in Eq. (6), the pp elastic scattering experimental data has been reproduced successfully in a tensor glueball exchange model[4]. The new form factor will be also applied to other diffractive processes[5]. However, it should be pointed out that not like the form factors used commonly, such as monopole, exponential and dipole form factor, the form of our new form factor in Eq. (6) is not unique, but it is singularity free form factor, the only one can be used to analyses involving channel crossing. It also give a most likely result of commonly used form factors for different purposes in different physical processes with a good stability as $\lambda_{s(t)}$ variation[6].

In conclusion, the dipole form factor given by Eq. (3) should not be used anymore in any calculations of the diffractive processes. The form factor with the form of Eq. (6) are strongly recommended to use in analysing diffractive processes.

References

1 Forshaw J R, Ross D A. QCD and the Pomeron, Cambridge University press, 1997

2 Pichowsky M A, Lee T–S H. Phys. Rev., 1997, **D56**: 1644; Kochelev N Z, Morri T, Vinnikoy A V. Phys. Lett., 1999, **B457**: 202; ZHAO Q, Diedelez J P, Guidal Met al. Nucl. Phys., 1999, **A660**: 323

3 Haider Q, LIU L. J. Phys. G: Nucl. Part. Phys., 1996, **22**: 1187

4 LIU L C, MA W X. J. Phys. G: Nucl. Part. Phys., 2000, **26**: L59

5 Bjorken J D. Phys. Rev., 1993, **D47**: 101

6 MA W X, LIU L C, Kisslinger L S. Pomeron Coupling to the Nucleon, 2001, to be submitted

一个新的、相对论性的、无奇异点的重子顶点形状因子*

马维兴[1]　梁伟红[1,2]　L.C. Liu[3]

1(中国科学院高能物理研究所　北京　100039)

2(广西师范大学物理与电子科学系　桂林　541004)

3 (Theoretical Division, Los Alamos National Laboratory, Los Alamos, N.M. 87545, USA)

摘要　提出了交叉道研究中的一种新的、相对论性的、无奇异性的顶角形状因子. 使用该形状因子的散射振幅具有从 t 道到 s 道或从 s 道到 t 道的交叉对称性.

关键词　形状因子　奇异性　交叉对称性

* 国家自然科学基金(19975053, 19835010, 10075081)资助