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非定域变换的广义 Ward 恒等式*

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摘要 基于高阶微商奇异拉氏量系统相空间 Green 函数的生成泛函，导出了该系统在定域和非定域变换下的广义正则 Ward 恒等式．对规范不变系统，从位形空间生成泛函出发，导出了该系统在定域、非定域和整体变换下的广义 Ward 恒等式．用于高阶微商非 Abel(Chern-Simons CS)理论，无需作出生成泛函中对正则动量的路径积分，即可导出正规顶角的某些关系．此外还给出了 BRS 变换下的 Ward-Takahashi 恒等式．

关键词 高阶微商场论 奇异拉氏量 路径积分 CS 理论

1 引言

场论中的高阶微商理论，可改善 Feynman 图的收敛性，受到人们的关注[1,2]．Ward 恒等式在量子场论中占重要地位，它是证明理论可重整化的工具；由它可导出诸 Green 函数间的关系．Ward 恒等式已有各种推广．从路径积分导出 Ward 恒等式的讨论中，通常是基于位形空间路径积分，这只适用于相空间路径积分对正则动量可积的情形(对含复杂约束的奇异拉氏量系统，往往很难或根本无法作出该积分)．相空间路径积分更基本，它适用于一般情形[3]．因此，从相空间路径积分出发研究系统的对称性具有更普遍的意义．在前面的工作中讨论了一阶微商系统[4]，这里来研究高阶微商系统．

本文基于相空间生成泛函，建立了高阶微商奇异拉氏量系统在定域和非定域变换下的广义正则 Ward 恒等式．对规范不变系统导出了位形空间中定域和非定域以及整体变换下的广义 Ward 恒等式．用于高阶微商非 Abel CS 理论，无需作出对正则动量的路径积分，就导出了正规顶角的一些关系；给出了 BRS 变换下的 Ward-Takahashi 恒等式．

2 非定域广义正则 Ward 恒等式

设场 $\varphi^{\alpha}(x)(\alpha=1,2,\cdots,n)$ 的运动由含高阶微商奇异拉氏量 $L[\varphi^{\alpha}_{(0)},\varphi^{\alpha}_{(1)},\varphi^{\alpha}_{(2)},\cdots]=$

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$\int d^3x \mathscr{L}(\varphi^a, \varphi^a_{,\mu}, \varphi^a_{,\mu\nu}, \cdots)$ 来描述[1]．由 Ostrogradsky 变换过渡到 Hamilton 描述时，$\varphi^a_{(s)}$ 的正则动量记为 $\pi^{(s)}_a$．该系统的正则变量 $\varphi^a_{(s)}, \pi^{(s)}_a$ 在相空间中存在固有约束，为广义约束 Hamilton 系统[1]．设 $\Lambda_k(\varphi^a_{(s)}, \pi^{(s)}_a) \approx 0\ (k = 1, 2, \cdots, K)$ 为系统的第一类约束，$\theta_i(\varphi^a_{(s)}, \pi^{(s)}_a) \approx 0\ (i = 1, 2, \cdots, I)$ 为第二类约束．按 Faddeev-Senjanovic(FS)路径积分量子化方案，该系统 Green 函数的相空间生成泛函为[5]

$$Z[j_a] = \int \mathscr{D}\varphi^a_{(s)} \mathscr{D}\pi^{(s)}_a \delta(\Phi) \sqrt{\det|\{\Phi, \Phi\}|} \exp\left\{ i\int d^4x (\mathscr{L}^P + j_a\varphi^a) \right\}, \tag{2.1}$$

其中 $\mathscr{L}^P = \pi^{(s)}_a \varphi^a_{(s+1)} - \mathscr{H}_C$，$\mathscr{H}_C$ 为正则 Hamilton 量密度．对第二类约束系统，$\{\Phi\}$ 代表所有第二类约束；对含第一类约束系统，$\{\Phi\}$ 代表所有约束和规范条件的总体．$\{\cdot,\cdot\}$ 代表广义 Poisson 括号，j_a 为 φ^a 的外源．利用 δ 函数和 Grassmann 变量 $C_l(x)$ 和 $C_k(x)$ 的积分性质，可将(2.1)式化为

$$Z[j_a, \eta_m, \xi_k, \xi_l] = \int \mathscr{D}\varphi^a_{(s)} \mathscr{D}\pi^{(s)}_a \mathscr{D}\lambda_m \mathscr{D}\bar{C}_k \mathscr{D}C_l \exp\left\{ i\int d^4x (\mathscr{L}^P_{\text{eff}} + j_a\varphi^a) \right\}, \tag{2.2}$$

其中

$$\mathscr{L}^P_{\text{eff}} = \mathscr{L}^P + \lambda_m\Phi_m + \frac{1}{2}\int d^4y \bar{C}^k(x)\{\Phi_k(x), \Phi_l(y)\} C_l(y), \tag{2.3}$$

而 $\lambda_m(x)$ 为乘子场，为简单起见，记 $\varphi^a_{(s)} = (\varphi^a_{(s)}, \lambda_m, \bar{C}_k, C_l)$，$J = (j_a, \eta_m, \xi_k, \xi_l)$，其中 η_m，ξ_k 和 ξ_l 分别为 λ_m，$\bar{C}_k$ 和 C_l 的外源．这样(2.2)式可写为

$$Z[J] = \int \mathscr{D}\varphi^a_{(s)} \mathscr{D}\pi^{(s)}_a \exp\left\{ i\int d^4x (\mathscr{L}^P_{\text{eff}} + J\varphi) \right\}. \tag{2.4}$$

定域规范不变性在场论中具有基本的意义，规范场论和共形场论中也讨论了非定域变换[4]．考虑增广相空间中的定域和非定域无穷小变换(省略 α 指标)：

$$\begin{cases} x'^{\mu} = x^{\mu} + \Delta x^{\mu} = x^{\mu} + R^{\mu}_{\sigma}\varepsilon^{\sigma}(x), \\ \varphi'_{(s)}(x') = \varphi_{(s)}(x) + \Delta\varphi_{(s)}(x) = \varphi_{(s)}(x) + A_{s\sigma}\varepsilon^{\sigma}(x) + \int d^4y E(x,y) B_{s\sigma}(y)\varepsilon^{\sigma}(y), \\ \pi^{(s)\prime}(x') = \pi^{(s)}(x) + \Delta\pi^{(s)}(x) = \pi^{(s)}(x) + U^s_{\sigma}\varepsilon^{\sigma}(x) + \int d^4y F(x,y) V^s_{\sigma}(y)\varepsilon^{\sigma}(y), \end{cases} \tag{2.5}$$

其中 $E(x,y)$ 和 $F(x,y)$ 为给定函数，R^{μ}_{σ}，$A_{s\sigma}$，$B_{s\sigma}$，U^s_{σ} 和 V^s_{σ} 为线性微分算符[4,6]，$\varepsilon^{\sigma}(x)(\sigma = 1, 2, \cdots, r)$ 为无穷小任意函数，它们及其微商的值在时空区域的边界上为零．在(2.5)式变换下，有效正则作用量的变更为[1,6]

$$\Delta I^P_{\text{eff}} = \Delta\int \mathscr{L}^P_{\text{eff}} d^4x = \int d^4x \left\{ \frac{\delta I^P_{\text{eff}}}{\delta\varphi_{(s)}}\delta\varphi_{(s)} + \frac{\delta I^P_{\text{eff}}}{\delta\pi^{(s)}}\delta\pi^{(s)} + D(\delta\pi^{(s)}\delta\varphi_{(s)}) + \partial_{\mu}[(\pi^{(s)}\varphi_{(s+1)} - \mathscr{H}_{\text{eff}})]\Delta x^{\mu} \right\}. \tag{2.6}$$

设(2.5)式正则变量变换的 Jacobi 行列式记为 $\bar{J}[\varphi_{(s)}, \pi^{(s)}, \varepsilon]$．生成泛函(2.4)在(2.5)式变换下的不变性，表明 $\delta Z/\delta\varepsilon^{\sigma}(x) = 0$，将(2.5)、(2.6)式代入(2.4)式，并对相应的项做分部积分，由 $\varepsilon^{\sigma}(x)$ 的边界条件，表面项积分为零．然后将生成泛函关于 $\varepsilon^{\sigma}(x)$ 求泛函微商，得

$$\left(J_\sigma^0 + \tilde{A}_{s\sigma}\left(\frac{\delta I_{\rm eff}^P}{\delta\varphi_{(s)}(x)}\right) + \tilde{U}_\sigma^s\left(\frac{\delta I_{\rm eff}^P}{\delta\pi^{(s)}(x)}\right) - \tilde{R}_\sigma^\mu\left[\varphi_{(s),\mu}\left(\frac{\delta I_{\rm eff}^P}{\delta\varphi_{(s)}(x)}\right) + \varphi_{,\mu}J + \pi_{,\mu}^{(s)}\left(\frac{\delta I_{\rm eff}^P}{\delta\pi^{(s)}(x)}\right)\right] + \int{\rm d}^4y\left\{\tilde{B}_{s\sigma}\left[E(y,x)\left(\frac{\delta I_{\rm eff}^P}{\delta\varphi_{(s)}(y)}\right) + D(\pi^{(s)}(y)E(y,x))\right] + \tilde{B}_{0\sigma}[E(y,x)J(y)] + \tilde{V}_\sigma^s\left[F(y,x)\left(\frac{\delta I_{\rm eff}^P}{\delta\pi^{(s)}(y)}\right)\right]\right\}\right)_{\varphi\to\frac{1}{i}\frac{\delta}{\delta J}} Z[J] = 0, \tag{2.7}$$

其中$J_\sigma^0 = -{\rm i}\,\delta\bar{J}/\delta\varepsilon^\sigma(x)\big|_{\varepsilon^\sigma=0}$. 当变换的 Jacobi 行列式不依赖于 $\varepsilon^\sigma(x)$时,$J_\sigma^0=0$. 而 $\tilde{A}_{s\sigma}$, $\tilde{B}_{s\sigma}$, $\tilde{R}_\sigma^\mu$, $\tilde{U}_\sigma^s$和$\tilde{V}_\sigma^s$分别为$A_{s\sigma}$, $B_{s\sigma}$, R_σ^μ, U_σ^s和V_σ^s的伴随算符[1,6]. (2.7)式称为高阶微商奇异拉氏量系统在定域和非定域变换下的广义正则 Ward 恒等式. 当(2.5)式中 $E=F=0$ 时,(2.7)式化为定域变换下的广义正则 Ward 恒等式[4]. 将(2.7)式对外源 J^0 多次求泛函微商可得其它的广义 Ward 恒等式,从而得诸 Green 函数的关系. 这样导出结果的优点在于勿需作出生成泛函中对正则动量的路径积分.

3 规范不变系统

高阶微商定域(规范)变换下不变系统,为广义约束 Hamilton 系统[1],该系统的量子化,也可以用 Faddeev-Popov(FP)技巧通过路径(泛函)积分变换得到[5],其有效拉氏量为$\mathscr{L}_{\rm eff} = \mathscr{L} + \mathscr{L}_{\rm f} + \mathscr{L}_{\rm gh}$, $\mathscr{L}$ 为原始拉氏量, $\mathscr{L}_{\rm f}$ 为规范固定项, $\mathscr{L}_{\rm gh}$为鬼粒子项,位形空间生成泛函为[5]

$$Z[J] = \int\mathscr{D}\varphi\exp\left\{{\rm i}\int{\rm d}^4x(\mathscr{L}_{\rm eff} + J\varphi)\right\}, \tag{3.1}$$

其中 φ 代表所有场量, J 为相应的外源.

考虑无穷小定域和非定域变换:

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + R_\sigma^\mu\varepsilon^\sigma(x), \\ \varphi'(x') = \varphi(x) + \Delta\varphi(x) = \varphi(x) + A_\sigma\varepsilon^\sigma(x) + \int{\rm d}^3yE(x,y)B_\sigma(y)\varepsilon^\sigma(y), \end{cases} \tag{3.2}$$

其中 R_σ^μ, A_σ 和 B_σ 为线性微分算符, $\varepsilon^\sigma(x)$为任意函数,在区域边界上为零(包括各级微商). 在(3.2)式变换下,有效作用量 $I_{\rm eff}$的变更为[7](将 $I_{\rm eff}$简记为 I):

$$\delta I = \int{\rm d}^4x\left\{\frac{\delta I}{\delta\varphi}\left[(A_\sigma - \varphi_{,\mu}R_\sigma^\mu)\varepsilon^\sigma(x) + \int{\rm d}^4yE(x,y)B_\sigma(y)\varepsilon^\sigma(y)\right] + \partial_\mu\left[j_\sigma^\mu\varepsilon^\sigma(x) + \sum_{m=0}^{N-1}\prod^{\mu\nu(m)}\partial_{\nu(m)}\int{\rm d}^4yE(x,y)B_\sigma(y)\varepsilon^\sigma(y)\right]\right\}. \tag{3.3}$$

(3.3)式中对$\partial_\mu(j_\sigma^\mu\varepsilon^\sigma(x))$的积分化为表面项后为零. 将剩下的有关项分部积分,由 $\varepsilon^\sigma(x)$的边界条件,相应的表面项积分为零. 将(3.2)和(3.3)式代入(3.1)式,设(3.2)式变换的 Jacobi 行列式为 1,生成泛函(3.1)在(3.2)式变换下的不变性,表明$\delta Z[J]/\delta\varepsilon^\sigma(x)\big|_{\varepsilon^\sigma(x)=0}=0$,于是得

$$\left\{\tilde{A}_\sigma\left(\frac{\delta I}{\delta\varphi}+J\right)+\tilde{R}^\mu_\sigma\left[\varphi_{,\mu}\left(\frac{\delta I}{\delta\varphi}+J\right)\right]+\int \mathrm{d}^4 y\tilde{B}_\sigma\left[E(y,x)\frac{\delta I}{\delta\varphi}+\partial_\mu\left(\sum_{m=0}^{N-1}\prod^{\mu\nu(m)}\partial_{\nu(m)}E(y,x)\right)+J\right]\right\}Z[J] \tag{3.4}$$

其中 $\tilde{A}_\sigma$, $\tilde{R}^\mu_\sigma$ 和 $\tilde{B}_\sigma$ 分别为 A_σ, R^μ_σ 和 B_σ 的伴随算符[1,6]. (3.4)式为高阶微商规范不变系统的生成泛函(3.1)式在定域和非定域变换下满足的广义 Ward 恒等式.

现讨论无穷小整体变换:

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + \varepsilon_\sigma \tau^{\mu\sigma}(x,\varphi,\varphi_{,\mu},\varphi_{,\mu\nu}\cdots), \\ \varphi'(x') = \varphi(x) + \Delta\varphi(x) = \varphi(x) + \varepsilon_\sigma \xi^{\mu\sigma}(x,\varphi,\varphi_{,\mu},\varphi_{,\mu\nu}\cdots), \end{cases} \tag{3.5}$$

其中 $\varepsilon_\sigma(\sigma=1,2,\cdots,r)$ 为无穷小任意函数, $\tau^{\mu\sigma}$ 和 ξ^σ 为 $x,\varphi,\varphi_{,\mu},\cdots$ 的函数. 假设有效作用量在(3.5)式变换下不变,且变换的 Jacobi 行列式为 1,生成泛函(3.1)式在(3.5)式变换下的不变性,有

$$Z[J] = \int \mathscr{D}\varphi\left\{1+\varepsilon_\sigma\int \mathrm{d}^4 x[J(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})+\partial_\mu(J\varphi\tau^{\mu\sigma})]\right\} = \left\{1+\varepsilon_\sigma\int \mathrm{d}^4 x\left[J\left(\xi^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\mathrm{i}\delta J}\right)+\partial_\mu\left(\tau^{\mu\sigma}J\frac{\delta}{\mathrm{i}\delta J}\right)\right]\right\}_{\varphi\to\frac{\delta}{\mathrm{i}\delta J}}Z[J]. \tag{3.6}$$

从而生成泛函(3.1)满足如下广义 Ward 恒等式

$$\int \mathrm{d}^4 x\left[J\left(\xi^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\mathrm{i}\delta J}\right)+\partial_\mu\left(\tau^{\mu\sigma}J\frac{\delta}{\mathrm{i}\delta J}\right)\right]_{\varphi\to\frac{\delta}{\mathrm{i}\delta J}}Z[J]=0. \tag{3.7}$$

将(3.7)式关于外源 J 求泛函微商,然后让外源为零,可得 Green 函数间的若干关系式.

4　高阶微商非 Abel CS 理论

CS 理论在量子 Hall 效应等方面有直接应用. (2+1)为非 Abel CS 项与物质场耦合的拉氏量为[2,8]

$$\mathscr{L} = -\frac{c^2}{4\pi}D_\rho F^a_{\mu\nu}D^\rho F^{a\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}\left(\partial_\mu A^a_\nu A^a_\rho + \frac{1}{3}f^a_{bc}A^a_\mu A^b_\nu A^c_\rho\right) + \mathrm{i}\bar{\psi}\gamma^\mu D_\mu\psi, \tag{4.1}$$

其中 D_μ 为协变微商, f^a_{bc} 为群的结构常数, Dirac γ - 矩阵为 $\gamma^0=\sigma^3$, $\gamma^1=\mathrm{i}\sigma^1$ $\gamma^2=\mathrm{i}\sigma^2$ (σ^i 为 Pauli 矩阵). 理论的规范不变性要求 $\kappa=\frac{n}{4\pi}$(n 为整数)[9]. 场量 A^a_μ, $\dot{A}a_\mu\equiv B^a_\mu$, ψ 和 $\bar{\psi}$ 的正则动量记为 P^μ_a, Q^μ_a, π 和 $\bar{\pi}$. 拉氏量(4.1)是奇异的,此约束 Hamilton 系统的相空间生成泛函为[8]

$$Z[J,\xi,\bar{\xi}] = \int \mathscr{D}u\delta(\Omega^a_i)\exp\left\{\mathrm{i}\int \mathrm{d}^3 x(\mathscr{L}^P + \lambda^a_i\Lambda^a_i + \bar{\lambda}\theta + \bar{\theta}\lambda - \partial^\mu\bar{C}^a D^a_{\mu b}C^b + J^\mu_a A^a_\mu + \bar{J}\psi + \bar{\psi}J + \bar{\xi}^a C^a + \bar{C}^a\xi^a)\right\}, \tag{4.2}$$

其中 $u=(A^a_\mu, B^a_\mu, \psi, \bar{\psi}, P^\mu_a, Q^\mu_a, \bar{\pi}, \pi, \lambda, \bar{\lambda}, C^a, \bar{C}^a)$, $\mathscr{L}^P = \dot{A}^a_\mu P^\mu_a + \dot{B}^a_\mu Q^\mu_a + \dot{\bar{\psi}}\bar{\pi} + \pi\dot{\psi} - \mathscr{H}_C$, $\mathscr{H}_C$ 为正则 Hamilton 密度, $\theta\approx 0$ 和 $\bar{\theta}\approx 0$ 为第二类约束, $\Lambda^a_i\approx 0(i=1,2,3)$ 为第一类约束,

$\Omega_i^a \approx 0(i=1,2,3)$为规范条件[8]．这里仅对场量 $A_\mu^a,\psi,\bar{\psi},C^a,\bar{C}^a$ 引入了外源[5]．

在量子水平上，I^P 和 $\mathscr{L}_{gh}$在下列变换下不变[4]：

$$\begin{cases} A_\mu^{a'}(x)=A_\mu^a(x)+D_{\sigma\mu}^a\varepsilon^\sigma(x),\ B_\mu^{a'}(x)=B_\mu^a(x)+\partial_0 D_{\sigma\mu}^a\varepsilon^\sigma(x), \\ C^{a'}(x)=C^a(x)+\mathrm{i}(T_\sigma)_b^a C^b(x)\varepsilon^\sigma(x), \\ \bar{C}^{a'}(x)=\bar{C}^a(x)-\mathrm{i}\bar{C}^b(x)(T_\sigma)_b^a\varepsilon^\sigma(x)+\dfrac{\mathrm{i}}{\Box}\partial_\mu[\bar{C}^b(x)(T_\sigma)_b^a\,\partial^\mu\varepsilon^\sigma(x)], \\ \psi'(x)=\psi(x)-\mathrm{i}(T_\sigma)\psi(x)\varepsilon^\sigma(x),\ \bar{\psi}'(x)=\bar{\psi}(x)+\mathrm{i}\bar{\psi}(x)(T_\sigma)\varepsilon^\sigma(x), \end{cases} \tag{4.3}$$

正则动量做相应的变换，其中 $T_\sigma(\sigma=1,2,\cdots,r)$为规范群的生成元．在(4.3)式变换下，设记 $\delta(\mathscr{L}_g+\mathscr{L}_m)=F_\sigma(u,\lambda)\varepsilon^\sigma(x)$，生成泛函(4.2)在(4.3)式变换下的不变性，有如下广义 Ward 恒等式：

$$\Big\{J_\sigma^0+\mathrm{i}F_\sigma-\mathrm{i}\,\partial_\mu J_\sigma^\mu+f_{\sigma c}^a J_a^\mu\frac{\delta}{\delta J_c^\mu}+\mathrm{i}\,\bar{J}_a(T_\sigma)_\beta^a\frac{\delta}{\delta\bar{J}_\beta}-\mathrm{i}J_a(T_\sigma)_\beta^a\frac{\delta}{\delta J_\beta}+\mathrm{i}\bar{\xi}_a(T_\sigma)_b^a\frac{\delta}{\delta\bar{\xi}_b}-$$

$$\mathrm{i}\xi_a(T_\sigma)_b^a\frac{\delta}{\delta\xi_b}+\mathrm{i}\,\partial^\mu\Big[\partial_\mu\Big(\xi_a\frac{1}{\Box}\Big)(T_\sigma)_b^a\frac{\delta}{\delta\xi_b}\Big]\Big\}Z[J,\bar{\xi},\xi]=0. \tag{4.4}$$

令 $Z[J,\xi,\bar{\xi}]=\exp\{\mathrm{i}W[J,\xi,\bar{\xi}]\}$，通过泛函 Legendre 变换，引入正规顶角的生成泛函

$$\Gamma[A_\mu^a,\psi,\bar{\psi},C^a,\bar{C}^a]=W[J_a^\mu,\bar{J},J,\bar{\xi},\xi]-\int\mathrm{d}^3x(J_a^\mu A_\mu^a+\bar{J}\psi+\bar{\psi}J+\bar{\xi}_aC^a+\bar{C}^a\xi_a), \tag{4.5}$$

于是(4.4)式化为

$$J_\sigma^0+\mathrm{i}F_\sigma+\mathrm{i}\,\partial_\mu\frac{\delta\Gamma}{\delta A_\mu^\sigma}-\mathrm{i}f_{\sigma c}^aA_\mu^c\frac{\delta\Gamma}{\delta A_\mu^a}-\mathrm{i}(T_\sigma)_\beta^a\psi_\beta\frac{\delta\Gamma}{\delta\psi^a}+\mathrm{i}(T_\sigma)_\beta^a\bar{\psi}_\beta\frac{\delta\Gamma}{\delta\bar{\psi}^a}-$$

$$\mathrm{i}C^a(T_\sigma)_b^a\frac{\delta\Gamma}{\delta C^b}+\mathrm{i}\bar{C}^a(T_\sigma)_b^a\frac{\delta\Gamma}{\delta\bar{C}^b}-\mathrm{i}\,\partial^\mu\Big[\partial_\mu\Big(\frac{\delta\Gamma}{\delta\bar{C}^a}\frac{1}{\Box}\Big)(T_\sigma)_b^a\bar{C}^b\Big]=0. \tag{4.6}$$

将(4.6)式关于 $\psi_\lambda(x_2)$和 $\bar{\psi}_\rho(x_3)$求泛函微商，然后让所有场为零，$A_\mu^a=\psi=\bar{\psi}=C^a=\bar{C}^a=\lambda=0$，由于 J_σ^0 与场量无关[4]，于是得

$$\partial_{x_1}^\mu\frac{\delta^3\Gamma[0]}{\delta\bar{\psi}_\rho(x_3)\delta\psi_\lambda(x_2)\delta A_\sigma^\mu(x_1)}=\delta(x_1-x_2)(T_\sigma)_\lambda^a\frac{\delta^2\Gamma[0]}{\delta\bar{\psi}_\rho(x_3)\delta\psi_a(x_1)}-$$

$$\delta(x_1-x_3)(T_\sigma)_\rho^a\frac{\delta^2\Gamma[0]}{\delta\psi_\lambda(x_2)\delta\bar{\psi}_a(x_1)}. \tag{4.7}$$

将(4.12)式关于 $\bar{C}^k(x_2)$和 $C^m(x_3)$求泛函微商，然后让所有场为零，得

$$\partial_{x_1}^\mu\frac{\delta^3\Gamma[0]}{\delta\bar{C}^k(x_2)\delta C^m(x_3)\delta A_\sigma^\mu(x_1)}+\delta(x_1-x_2)(T_\sigma)_k^b\frac{\delta^2\Gamma[0]}{\delta\bar{C}^b(x_1)\delta C^m(x_3)}-$$

$$\delta(x_1-x_3)(T_\sigma)_m^b\frac{\delta^2\Gamma[0]}{\delta\bar{C}^k(x_2)\delta C^b(x_1)}-$$

$$\partial^\mu\Big[\partial_\mu\Big(\frac{\delta^2\Gamma[0]}{\delta\bar{C}^a(x_1)\delta C^m(x_3)}\frac{1}{\Box}\Big)(T_\sigma)_k^a\delta(x_1-x_2)\Big]=0. \tag{4.8}$$

这里导出正规顶角的 Ward 恒等式与传统方法不同的显著优点在于勿需作出相空间路径

积分中对正则动量的积分. 对约束结构复杂的系统,作出该积分是很困难的,甚至是不可能的. 此外变换(4.3)是线性(非定域)的,导出上述结果仅要求变换保持 I^P 和 $\mathscr{L}_{gh}$ 在理论中不变,这些都与 BRS 变换不同.

现在考虑位形空间中的 BRS(整体)变换

$$\begin{cases}\delta A_\mu^a = -\tau D_{\mu b}^a C^b,\\ \delta\psi = \mathrm{i}\tau C^b T^b\psi, \delta\bar{\psi} = -\mathrm{i}\tau\bar{\psi}C^b T^b,\\ \delta C^a = \frac{1}{2}\tau f^a_{\ bc}C^b C^c, \delta\bar{C}^a = -\partial^\mu A_\mu^a,\end{cases} \tag{4.9}$$

其中 τ 是 Grassmann 参量. 不难验证, $\delta(D_{\mu b}^a C^b) = \delta(\delta\psi) = \delta(\delta\bar{\psi}) = \delta(\delta C^a) = 0$. 对 δA_μ^a, δC^a, $\delta\psi$ 和 $\delta\bar{\psi}$ 分别引入外源 u_a^μ, v^a, $\bar{\eta}$ 和 η,给出扩展生成泛函

$$Z[J,\bar{\xi},\xi,u,v,\eta,\bar{\eta}] = \int\mathscr{D}A_\mu^a\mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}\bar{C}^a\mathscr{D}C^a\exp\Big\{\mathrm{i}\int\mathrm{d}^3x(\mathscr{L}_{\rm eff} + J_a^\mu A_\mu^a + \bar{J}\psi + \bar{\psi}J + \bar{\xi}_a C^a + \bar{C}^a\xi_a + u_a^\mu\delta A_\mu^a + v_a\delta C^a + \bar{\eta}\delta\psi + \delta\bar{\psi}\eta)\Big\}, \tag{4.10}$$

其中 $\mathscr{L}_{\rm eff}$是在 Lorentz 规范下用 FP 技巧得到的有效拉氏量,它在(4.9)式变换下不变,且变换的 Jacobi 行列式为 1. 生成泛函(4.10)在(4.9)式变换下不变,就有

$$\int\mathscr{D}A_\mu^a\mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\bar{C}^a\mathscr{D}C^a\Big[\int\mathrm{d}^3x[J_a^\mu\delta A_\mu^a + \bar{J}\delta\psi + \delta\bar{\psi}J + \bar{\xi}_a\delta C^a + \delta\bar{C}^a\xi_a]\exp\Big\{\mathrm{i}\int\mathrm{d}^3x(\mathscr{L}_{\rm eff} + J_a^\mu A_\mu^a + \bar{J}\psi + \bar{\psi}J + \bar{\xi}_a C^a + \bar{C}^a\xi_a + u_a^\mu\delta A_\mu^a + v_a\delta C^a + \bar{\eta}\delta\psi + \delta\bar{\psi}\eta)\Big\} = 0. \tag{4.11}$$

从而生成泛函(4.10)满足如下广义 Ward 恒等式:

$$\int\mathrm{d}^3x\Big[\bar{J}\frac{\delta}{\delta\bar{\eta}} + J\frac{\delta}{\delta\eta} + J_a^\mu\frac{\delta}{\delta u_a^\mu} + \bar{\xi}_a\frac{\delta}{\delta v_a} - \xi_a\partial^\mu\Big(\frac{\delta}{\delta J_a^\mu}\Big)\Big]Z[J,\bar{\xi},\xi,u,v,\eta,\bar{\eta}] = 0. \tag{4.12}$$

此结果也可以从生成泛函(4.2)出发导出. 生成泛函和 $\mathscr{L}_{\rm eff}$在(4.9)式变换下的不变性,可导出量子水平的 BRS 守恒荷[4].

5 结论和讨论

对约束 Hamilton 系统,FS 相空间路径积分量子化比直观的 FP 位形空间路径积分量子化更基本,后者仅适用于规范不变系统. 对某些场论模型,将 FS 量子化结果作出对正则动量的路径积分后,可化为 FP 量子化的结果. 本文分别用 FS 和 FP 方案导出了高阶微商奇异拉氏量系统在非定域变换下的 Ward 恒等式,用于非 Abel CS 理论,不难验证,按 FS 方案和 FP 方案可导致同样结果,表明 FP 方案对此模型适用.

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Generalized Ward Identities for Non-local Transformation*

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Abstract Based on the phase-space generating functional of Green function for a system with a singular higher-order Lagrangian, the generalized canonical Ward identities under the local and non-local transformation in phase space for such a system have been derived. Starting from the configuration-space generating functional for a gauge-invariant system, the generalized Ward identities were deduced under the local, non-local and global transformation, respectively. The applications to the non-Abelian Chern-Simons theories with higher derivatives were given. Some relationships among the proper vertices have been deduced, in which one does not need to carry out the integration over canonical momenta in phase-space generating functional. The Ward-Takahashi identities for BRS transformation are also obtained.

Key words field theories with higher derivatives, singular Lagrangian, path integral, Chern-Simons theories

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