Several Crucial Problems in Evaluating Spectra of Baryons with Two Heavy Quarks *

TONG ShengPing[1] DING YiBing[1] GUO XinHeng[2,3] JIN HongYing[4],
LI XueQian[5] SHEN PengNian[3] ZHANG Rui[5]

1 (*Graduate School, University of Science and Technology of China at Beijing, The Chinese Academy of Sciences, Beijing* 100039, *China*)

2 (*Department of Physics and Mathematical Physics, and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, SA* 5005, *Australia*)

3 (*Institute of High Energy Physics, The Chinese Academy of Sciences, Beijing* 100039, *China*)

4 (*Department of Physics, University of Mainz, Germany*)

5 (*Department of Physics, Nankai University, Tianjin* 300071, *China*)

Abstract The spectra of baryons which include two heavy quarks and one light quark can be treated as a two-body system, where two heavy quarks constitute a bosonic diquark. We derive the effective potential between the light quark and the heavy diquark. In this work we have discussed several serious problems: (1) the operator ordering, (2) the errors caused by the non-relativistic expansion, (3) spin-spin coupling and (4) the mixing between baryon states with scalar-diquark and vector-diquark.

Key words baryon mass, quark-diquark model, nonrelativistic expansion, operator ordering

The diquark structure in baryons has received increasing interests from high energy theorists[1]. It is indeed dubious for the baryons which only contain light quarks, because of the spatial dispersion of the light quarks[2,3]. If there are two heavy quarks (bb, bc, cc) in a baryon, they tend to constitute a substantial diquark with small spatial size and serve as a static $\bar{3}$ color source for the light quark[4,5]. Recently, Ebert et al. evaluated the spectra of such baryons in terms of the local Schrödinger-like quasi-potential equation[6]. Similarly, Gershtein et al. estimated the spectroscopy of doubly charmed baryons where angular and radial excited states are included[7].

In the potential model, the interaction between the light quark and the heavy diquark can be derived by calculating their elastic scattering amplitudes[8]. The key point is the form of the effective vertices of the diquark-gluon interaction. In this work, by retaining k^2-dependence explicitly, we rederive the effective potential by using the Bethe-Salpeter (B-S) equation and obtain the effective vertex. In calculating the baryon mass, we notice several serious problems which have not carefully been discussed in earlier literature[6,2]. (1) A systematic way of deriving the form factor at the diquark-gluon vertex. (2) The operator ordering problem in Fourier transforming scattering amplitudes in momentum space into configuration space. (3) Intolerable errors caused by the relativistic effect due to the existence of light quark. The parameters obtained by fitting the data of J/ψ, Υ and etc.

Received 5 January 2000
* Supported by National Natural Science Foundation of China, Major Project of The Chinese Academy of Sciences and The Australian Research Council.
691-696

should not be valid to the cases where light flavors are involved. (4)Due to the $\delta^3(\mathbf{r})$ behavior of the spin - spin coupling, the diquark picture may turn down. (5)Possible mixing between baryon states where the bc - diquarks are scalar and axial vector respectively. In this short paper, we mainly discuss these problems.

1 Effective Vertex of Diquark - Gluon Coupling

Since the diquark is not rigorously a point - like subject, we cannot employ commonly used vertex. Instead, we derive such effective vertex from the B - S equation where the kernel is chosen to be the Cornell potential in analog to the meson case. The effective vertex DD′g, where D and D′ could be either scalar diquark (S) or axial - vector diquark(A), and g denotes the gluon, are

$$\langle S'(v')|J_\mu|S(v)\rangle=\sqrt{MM'}[f_1 v'_\mu+f_2 v_\mu],\ \text{for SSg coupling,} \tag{1}$$

$$\begin{aligned}\langle A'(v',\eta')|J_\mu|A(v,\eta)\rangle=\sqrt{MM'}[&f_3(\eta\cdot\eta'^*)v'_\mu+f_4(\eta'^*\cdot\eta)v_\mu+\\&f_5(\eta\cdot v')(\eta'^*\cdot v)v'_\mu+f_6(\eta\cdot v')(\eta'^*\cdot v)v_\mu+\\&f_7\eta'^*_\mu(\eta\cdot v')+f_8(\eta'^*\cdot v)\eta_\mu+\\&f_9 i\epsilon_{\mu\nu\rho\sigma}\eta'^{*\nu}\eta^\rho v'^\sigma+\\&f_{10}i\epsilon_{\mu\nu\rho\sigma}\eta'^{*\nu}\eta^\rho v^\sigma],\ \text{for AAg coupling,}\end{aligned} \tag{2}$$

$$\begin{aligned}\langle A'(v',\eta')|J_\mu|S(v)\rangle=\sqrt{MM'}[&f_{11}\eta'^*_\mu+f_{12}(\eta'^*\cdot v)v'_\mu+\\&f_{13}(\eta'^*\cdot v)v_\mu+f_{14}i\epsilon_{\mu\nu\rho\sigma}\eta'^{*\nu}v'^\rho v^\sigma],\ \text{for ASg coupling,}\end{aligned} \tag{3}$$

where $v', v, \eta', \eta, M', M$ are the four - velocities, polarization vectors(for axial - vector diquark only), and masses of diquarks in the "final" and "initial" states of the scattering, respectively. The corresponding form factors are derived by solving the B - S equation and the details were given in our previous work[5]. In our case, by simply analyzing the parity and keeping terms up to order $\boldsymbol{P}^2$ in the nonrelativistic expansion, we can fix the relations of all the f_i's.

Because form factors f_i's involve B - S integrals, they usually cannot be analytically derived. In order to serve our final goal, we simulate the numerical results by the expression

$$f=\frac{(A+B)\boldsymbol{k}^2+AC^2)}{(\boldsymbol{k}^2+C^2)}\cdot\frac{1}{1+\frac{\boldsymbol{k}^2}{\Lambda^2}}, \tag{4}$$

where $\boldsymbol{k}$ is the exchanged three - momentum, and k the four - momentum with $k_0=0$, and obtain the numerical values of parameters A, B, C and Λ. This is the familiar pole - like form factor which is widely used in phenomenology[3].

2 Ordering of Operator

The effective potential is derived by calculating the elastic scattering amplitude. Then, turning $\eta(\eta')$ into quantum mechanics(QM) spin - operator, we have

$$\eta=\frac{1}{\sqrt{2}}((\vec{\beta}\cdot\boldsymbol{S})\gamma\boldsymbol{S}+\frac{\gamma-1}{\vec{\beta}^2}(\vec{\beta}\cdot\boldsymbol{S})\vec{\beta}), \tag{5}$$

where $\boldsymbol{S}$ is the spin - operator, $\vec{\beta}=\boldsymbol{P}/M$, $\gamma=E/M$ are the boost factors and $\boldsymbol{P}, E, M$ are the momentum, energy and mass, respectively. When we transform them into the QM operators and Fourier - transforming them into configuration space, the ordering problem generally appears.

We consider the following four different orders for $\boldsymbol{P}, \boldsymbol{P}, g(r)$:

Ordering 1	Ordering 2	Ordering 3	Ordering 4
$g(\boldsymbol{r})\boldsymbol{p}^2$	$\frac{1}{2}(g(\boldsymbol{r})\boldsymbol{p}^2+\boldsymbol{p}^2 g(\boldsymbol{r}))$	$\boldsymbol{p}\cdot g(\boldsymbol{r})\boldsymbol{p}$	$\frac{1}{4}[g(\boldsymbol{r})\boldsymbol{p}^2+2\boldsymbol{p}\cdot g(\boldsymbol{r})\boldsymbol{p}+\boldsymbol{p}^2 g(\boldsymbol{r})]$

where $g(r)$ is a function of $r(=|\boldsymbol{r}|)$ and only last three terms are Hermitian.

It should be mentioned that the expression $g(r)\,\hat{\boldsymbol{P}}^2$ is not Hermitian. It can be proved that if all the quantities are real, there is no angular - dependence and $g(r)$ is less singular than $1/r$, the non - hermiticity would not explicitly show up. In our case, non - hermiticity would present (refer to our result below).

3 Potential in Configuration Space - New Treatment of the Diquark Picture

The potential finally can be written as $V = V_{gluon} + V_{conf}$ with $V_{conf} = V^S_{conf} + V^V_{conf}$. Here the superscripts S and V denote the vector and scalar parts of the confinement. The single gluon exchanged potential V_{gluon} has the following form:

$$V_{gluon} = V'^{SS,AA,SA}_{gluon} + V_{spin}, \tag{6}$$

where the explicit forms of $V'^{SS,AA,SA}_{gluon}$ can be found in Ref, [9] and V_{spin} is the spin - spin coupling term with a coefficient proportional to $\delta^3(\boldsymbol{r})$. Due to $\delta^3(\boldsymbol{r})$, V_{spin} is short - range behaviored and the light quark only sees one heavy quark each time, but the whole diquark. Instead of the strict three - body treatment on $g(M_D)\delta_3(\boldsymbol{r})$, we introduce a simple phenomenological means

$$V_{spin} = g(m_{Q'})\delta^3(\boldsymbol{r}-\boldsymbol{r}'_2)\boldsymbol{S}_1\cdot\boldsymbol{S}'_2 + g(m_{Q''})\delta^3(\boldsymbol{r}-\boldsymbol{r}''_2)\boldsymbol{S}_1\cdot\boldsymbol{S}''_2, \tag{7}$$

where $g(M_D)$, $g(m_{Q'})$ and $g(m_{Q''})$ are the functions of the masses of the diquark, heavy quarks 1 and 2, respectively, say

$$g(M) \propto \frac{1}{m_q m_Q},$$

and $\boldsymbol{r}-\boldsymbol{r}'_2$ and $\boldsymbol{r}-\boldsymbol{r}''_2$ are the distance vectors of the light quark to the corresponding heavy quarks. Thus, the light quark interacts with one heavy quark only, while leaves another heavy quark as a spectator. Taking $V'^{SS,AA}_{gluon} + V_{conf}$ as the 0 - th order potential and treating V_{spin} as a perturbation, these two spin - spin terms lead to an extra contribution

$$\Delta E_{spin} = g(m_{Q'})\langle \boldsymbol{S}_1\cdot\boldsymbol{S}'_2\rangle |\Psi_{qQ'}(0)|^2 + g(m_{Q''})\langle \boldsymbol{S}_1\cdot\boldsymbol{S}''_2\rangle |\Psi_{qQ''}(0)|^2, \tag{8}$$

where each term is proportional to the square of the corresponding wavefunction at the origin $\Psi_{qQ}(0)$. Similar to that in the parton model in deep inelastic scattering (DIS), the wavefunction of the spectator can be normalized away.

In analog to the analysis of Falk et al.[4], $\Psi_{qQ}(0)|^2$ can be obtained by $|\Psi_{qQ}(0)|^2$ or $|\Psi_{q\bar{Q}}(0)|^2$ except a color - suppression factor 1/8, as $\Psi(0)$ determined mainly by the Coulomb potential is proportional to $(C_F\alpha_s)^{3/2}$.

It is true that in S - wave meson, the splitting between M and M^* is caused by the spin - spin interaction only, we can use the well - measured mass splittings $M_{D^*} - M_D$, $M_{B^*} - M_B$ and $M_{D_s^*} - M_{D_s}$ as inputs to deduce the spin - coupling contribution to the baryon with two heavy quarks. Substituting this relation into ΔE_{spin}, we have

$$\Delta E_{spin} = \langle \boldsymbol{S}_1\cdot\boldsymbol{S}'_2\rangle_B \left(\frac{M^*_{M'} - M_{M'}}{8\langle \boldsymbol{S}_1\cdot\boldsymbol{S}'_2\rangle_M}\right) + \langle \boldsymbol{S}_1\cdot\boldsymbol{S}''_2\rangle_B \left(\frac{M^*_{M''} - M_{M''}}{8\langle \boldsymbol{S}_1\cdot\boldsymbol{S}''_2\rangle_M}\right), \tag{9}$$

where M′ and M″ are the masses of corresponding mesons, respectively, and the subscripts B and M denote that the matrix elements of the spin - coupling are taken for baryon and meson, respectively. Meanwhile we need to keep the total spin of the two - heavy - quark system (diquark)

$$\mathbf{S}_2 = \mathbf{S}'_2 + \mathbf{S}''_2$$

to be 0 (scalar) or 1 (axial vector) so that $\langle \mathbf{S}_1\cdot\mathbf{S}'_2(\mathbf{S}''_2)\rangle$ can straightforwardly be calculated.

Obviously, for the diquark composed of two different heavy quarks, $g(m_{Q'}) \neq g(m_{Q''})$ and $|\Psi_{qQ'}(0)| \neq |\Psi_{qQ''}(0)|$, two components in V_{spin} would make different contributions.

It is noticed that the data of B_s^* are not available. It is also noted that V_{spin} is inversely proportional to $m_q m_Q$ and the mass splittings are 45.7 MeV for B^* and B and 142.12 MeV for D and D^*, namely their ratio is roughly $45.7/142.12 \approx m_c/m_b \approx 1.55/4.88$. Therefore, we can reasonably assume

$$M_{B_s^*} - M_{B_s} = \frac{m_c}{m_b}(M_{D_s^*} - M_{D_s}).$$

4 Choice of Parameters

The variational method is employed to evaluate the 0-th order spectra, and V_{spin} is served as the perturbative term to get the fine splitting. The advantage of using the variational method is that we are able to treat all terms simultaneously, so that the large relativistic correction problem in the perturbative method can be avoided[10]. Because of existence of a light quark in baryon, unlike the heavy quarkonium, such as J/ψ, Υ etc., truncating the nonrelativistic expansion to the order of P^2/m^2 is not a good approximation. In the process of fitting the data of mesons containing a light quark, such as $B^{(*)}$, we attribute uncertain factors into phenomenological parameters α_s, a and b which are not directly measurable. The A, B, C-values in Eq.(4) are

$A=1.00$, $B=-1.00$, $C=3.11$ GeV, $\Lambda=2.86$ GeV, for cc diquark;
$A=1.00$, $B=-1.00$, $C=8.30$ GeV, $\Lambda=6.45$ GeV, for bc diquark;
$A=1.00$, $B=-1.00$, $C=5.09$ GeV, $\Lambda=4.33$ GeV, for bb diquark.

The constituent quark masses and the heavy diquark masses are taken as[2]

$$m_u = m_d = 0.33\text{ GeV},\ m_s = 0.5\text{ GeV},\ m_c = 1.55\text{ GeV},\ m_b = 4.88\text{ GeV},$$
$$M_{cc} = 3.26\text{ GeV},\ M_{bc} = 6.52\text{ GeV},\ M_{bb} = 9.79\text{ GeV}.$$

It is noted that bb and cc diquark must be axial vectors, but bc can be either a scalar or an axial vector, the mass splitting of the scalar and axial-vector bc diquarks can be neglected in practical calculations.

In the calculation $\kappa = 0.5$ is adopted. The resultant baryon spectra with various ordering schemes are given in Table 1. It should be mentioned that changing the value of κ would not cause

Table 1 The baryon mass in various ordering schemes.

type	Ordering 1	Ordering 2	Ordering 3	Ordering 4
(ccq)(1/2)	3.720	3.733	3.703	3.719
(ccq)(3/2)	3.838	3.852	3.823	3.838
(ccs)(1/2)	3.723	3.767	3.726	3.745
(ccs)(3/2)	3.817	3.889	3.841	3.867
$(cbq)(1/2)_S$	7.057	7.062	7.063	7.063
$(cbq)(1/2)_A$	7.016	7.010	7.000	7.005
(cbq)(3/2)	7.094	7.088	7.095	7.092
$(cbs)(1/2)_S$	7.048	7.089	7.077	7.083
$(cbs)(1/2)_A$	6.997	7.037	7.010	7.023
(cbs)(3/2)	7.076	7.116	7.109	7.113
(bbq)(1/2)	10.314	10.299	10.289	10.293
(bbq)(3/2)	10.352	10.337	10.329	10.332
(bbs)(1/2)	10.296	10.326	10.299	10.312
(bbs)(3/2)	10.336	10.336	10.341	10.353

In the table, q = u or d, A and S stand for the axial vector and scalar, respectively. In the cacultion, $\kappa = 0.5$; in the ordering 1 case, $\alpha_s = 0.65$, $a = 0.12$, $b = -0.355$; in the ordering 2 case, $\alpha_s = 0.61$, $a = 0.13$, $b = -0.33$; in the ordering 3 case, $\alpha_s = 0.61$, $a = 0.14$, $b = -0.375$; and in the ordering 4 case, $\alpha_s = 0.58$, $a = 0.15$, $b = -0.415$.

serious deviation on result. For instance, in the ordering 2 case, when κ changes from 0 to 1, the mass deviation is about 20—30 MeV, which can be found in Ref. [9]. It is also noted that in Ebert et al.'s work, they used $\kappa = -1$[6].

As a summary, we clarify several serious problems existing in the earlier calculation for baryons containing two heavy quarks.

Based on the B-S equation, we derive the effective vertex DD′g so that the spatial dispersion of the diquark can be compensated by the form factor.

The operator ordering problem is carefully studied. It is found that different ordering schemes would lead to the different values of parameters α_s, a and b which are not directly measurable. As a result, the masses of baryons only deviate a few MeV to a few tens of MeV as long as the hermiticity of the operator is respected, but it would go up to 50 MeV if the hermiticity does not hold.

Because of serious relativistic effects in the baryon where a light flavor is involved, the variational method has superiority to the perturbative method. The $B^{(*)}$ data is also adopted in fixing parameters so that uncertainties and errors brought up by the truncation in the nonrelativistic expansion can mostly be eliminated.

To keep the diquark picture valid, the spin-spin interaction term with coefficient $\delta^3(\boldsymbol{r})$ is separated from other parts. It can be decomposed into two pieces and each piece only concerns one of the heavy quarks.

Eventually, the derived form factor further polishes the diquark picture. But, some small deviations from reality, including the diquark masses, still exist and should be further investigated.

Finally, we point that although the mixing term derived in quantum field theory is not trivially zero, but in QM the off-diagonal matrix elements for the states with S and A diqaurks are exact zero:

$$\langle\psi(1/2,A,l=0)|V^{(SA)}_{gluon}|\psi(1/2,S,l=0)\rangle = \langle\psi(1/2,A,l=1)|V^{(SA)}_{gluon}|\psi(1/2,S,l=1)\rangle \equiv 0. \tag{10}$$

However, such mixing should exist and may play important roles in hadronic spectra. For instance, the mixing between glueball and quarkonium is known as very important or even crucial in phenomenology. Therefore, the mixing effect in the baryonic diquark picture should be future investigated[11].

The B-factory and other facilities of high energy experiments may provide the data of $\Xi_{cc}^{(*)}$ and the others such baryons. Once the data are available, we may readjust our input parameters and predict the spectra and other characters of baryons again. It would provide a chance to further testify the validity of the baryonic diquark picture and the nonrelativistic potential model.

References

1 Anselmino M et al. Rev. Mod. Phys., 1993, **65**:1199.

2 Fredriksson S, Hansson J, Ekelin S. Z. Phys., 1997, **C75**:107

3 Anselmino M, Kroll P, Pire B. Z. Phys., 1987, **C36**:89; Guo X, Li X, Commun. Theor. Phys., 1994, **22**:337; GUO X, Muta T, Phys. Rev., 1996, **D54**:4629; Guo X, Thomas A W, Williams A G. Phys. Rev., 1998, **D59**:116007

4 Falk A, Luke M, Savage M. Phys. Rev., 1994, **D49**: 555

5 GUO X, JIN H, LI X, Phys. Rev., 1996, **D53**:1153; Guo X, Jin H, Li X, Phys. Rev. 1998, **D58**:114007

6 Ebert D, Fanetov F, Galkin V. Z. Phys., 1997, **C76**:111

7 Gershtein S, Kiselev V, Likhoded A. DESY 98-080

8 Berestetskii V, Lifshitz E, Pitaevskii L. Quantum Electrodynamics, Pergamon Press, Oxford, 1980

9 DING Y et al. hep-ph/9910259

10 DING Y, LI X, SHEN P. Phys. Rev., 1999, **D60**:074010

带有两个重夸克的重子谱计算中几个困难问题的研究*

童胜平[1] 丁亦兵[1] 郭新恒[2,3] 金宏英[4]
李学潜[5] 沈彭年[3] 张 瑞[5]

1(中国科学院中国科学技术大学研究生院 北京 100039)
2 (Department of Physics and Mathematics Physics, and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, SA 5005, Australia)
3(中国科学院高能物理研究所 北京 100039)
4(Department of Physics, University of Mainz, Germany)
5(南开大学物理系 天津 300071)

摘要 由两个重夸克和一个轻夸克组成的重子可以看作是一个两体系统.它的两个重夸克组成一个玻色型的双夸克团.利用B－S方程导出了它的轻夸克和重的双夸克之间的等效相互作用势.在利用这种势计算重子质量的过程中,发现有几个困难问题需要深入探讨.它们是:(1)算符排序,(2)由非相对论展开带来的误差,(3)自旋－自旋耦合,(4)在标量双夸克组成的重子态和矢量双夸克组成的重子态之间的混合.本文详细地讨论并适当地处理了这些问题.

关键词 重子质量 夸克－双夸克模型 非相对论展开 算符排序

2000－01－05 收稿
* 国家自然科学基金,中国科学院重大基金和澳大利亚研究委员会基金部分资助