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Collective Motion of a pure Octupole Deformed System *

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Abstract The collective motion of a pure octupole deformed system is treated as the vibrations in body-fixed frame and rotation of this system about the axes of lab-system, as well as the coupling between vibrations and rotation. The quantized operator of kinetic energy is derived and the collective spectra built on some special equilibrium shapes are discussed.

Key words octupole deformation, collective motion, body-fixed frame

1 Introduction

It's well known that the quadruple deformation is by far the most important deformation in the nuclear physics and the collective motion of quadruple deformed nuclei was treated as the vibrations in body-fixed frame(β_2 and γ_2 vibrations) and rotation of this system about the axes of lab-frame[1]. However, the higher multipole deformation is also essential for a satisfactory description of nuclear properties. There are attempts of generalization of the Bohr's collective Hamiltonian to pure octupole deformed system[2—4]. But because of the misleading definition of the body-fixed frame this problem has not been solved. In recent years, the collective spectra of pure octupole deformed system have been paid attention. The collective bands of octupole deformation have become one of the most heating frontier topic (Refs. [5, 6] for example). People have already discussed the parameterization of octupole deformation and some body-fixed frames of octupole deformation. In Ref. [7], 18 various body-fixed frames are defined, two of them are found to hold the simplest determinant and thought to be the most convenient. In the following, we will derive the quantized operator of kinetic energy in the body-fixed frame defined by deformation parameters $a_{30}, a_{31}, a_{32}, b_{31}$.

2 Quantization of the Collective Kinetic Energy

There is no unique way to perform this quantization but, as in the quadruple case, the Pauli prescription can be used. This recipe is designed to give the right answer when the generalized variables are transformed to Cartesian coordinates. Given a classical Hamiltonian written in terms of variables $a_{30}, a_{31}, a_{32}, b_{31}, \phi_1, \phi_2, \phi_3$, referred to as q_i, and their time derivatives, with a kinetic energy given in Ref. [7]. The Pauli prescription replaces the kinetic energy by the operator.

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$$T = -\frac{\hbar^2}{2B_3}\sum_{mn}\frac{1}{\sqrt{g}}\frac{\partial}{\partial q_m}\sqrt{g}G^{-1}_{mn}\frac{\partial}{\partial q_n}. \tag{1}$$

We obtain

$$T = T_{\mathrm{vib}} + T_{\mathrm{rot}} + T_{\mathrm{cou}}, \tag{2}$$

$$T_{\mathrm{vib}} = -\frac{\hbar^2}{2B_3}\frac{1}{3a_{32}^3}\sum_{i,j=1}^{4}\frac{\partial}{\partial q_i}X_{ij}\frac{\partial}{\partial q_j}, \tag{3}$$

$$T_{\mathrm{cou}} = -\frac{\hbar^2}{2B_3}\frac{1}{3a_{32}^3}\sum_{i=1}^{4}\sum_{j=1}^{3}\left\{Y_{ij},\frac{\partial}{\partial q_i}\right\}\frac{\partial}{\partial \phi_j},$$

$$T_{\mathrm{rot}} = -\frac{\hbar^2}{2B_3}\frac{1}{3a_{32}^3}\sum_{i,j=1}^{3}Z_{ij}\frac{\partial}{\partial \phi_i}\frac{\partial}{\partial \phi_j},$$

here $X_{ij} = X_{ji}$ and

$$\begin{aligned}
X_{11} &= 3a_{32}(4a_{31}^2 + a_{32}^2 + 4b_{31}^2),\\
X_{12} &= 2a_{31}(-6a_{30}a_{32} + \sqrt{15}a_{32}^2 - \sqrt{15}b_{31}^2),\\
X_{13} &= 2\sqrt{15}a_{32}(-a_{31}^2 + b_{31}^2),\\
X_{14} &= 2b_{31}(-6a_{30}a_{32} + \sqrt{15}a_{31}^2 - \sqrt{15}a_{32}^2),\\
X_{22} &= (48a_{30}^2a_{32}^2 - 16\sqrt{15}a_{30}a_{32}^3 + 32a_{32}^4 + 5a_{31}^2b_{31}^2 +\\
&\quad 8\sqrt{15}a_{30}a_{32}b_{31}^2 - 17a_{32}^2b_{31}^2 + 5b_{31}^4)/(4a_{32}),\\
X_{23} &= (2\sqrt{15}a_{30} - 5a_{32})a_{31}a_{32},\\
X_{24} &= a_{31}b_{31}(-5a_{31}^2 + 17a_{32}^2 - 5b_{31}^2)/(4a_{32}),\\
X_{33} &= a_{32}(5a_{31}^2 + 3a_{32}^2 + 5b_{31}^2),\\
X_{34} &= -(2\sqrt{15}a_{30} + 5a_{32})a_{32}b_{31},\\
X_{44} &= (48a_{30}^2a_{32}^2 + 16\sqrt{15}a_{30}a_{32}^3 + 32a_{32}^4 + 5a_{31}^2b_{31}^2\\
&\quad - 8\sqrt{15}a_{30}a_{32}a_{31}^2 - 17a_{31}^2a_{32}^2 + 5a_{31}^4)/(4a_{32}),
\end{aligned} \tag{6}$$

$$Y = \begin{pmatrix}
\sqrt{24}a_{32}b_{31} & -\sqrt{24}a_{31}a_{32} & -2\sqrt{15}a_{31}b_{31}\\
-\sqrt{\frac{5}{2}}a_{31}b_{31} & \sqrt{\frac{5}{2}}b_{31}^2 + \sqrt{24}a_{30}a_{32} - \sqrt{10}a_{32}^2 & \frac{b_{31}}{4a_{32}}(5a_{31}^2 + 4\sqrt{15}a_{30}a_{32} - 7a_{32}^2 + 5b_{31}^2)\\
\sqrt{10}a_{32}b_{31} & \sqrt{10}a_{31}a_{32} & 0\\
\sqrt{\frac{5}{2}}a_{31}^2 - \sqrt{24}a_{30}a_{32} - \sqrt{10}a_{32}^2 & -\sqrt{\frac{5}{2}}a_{31}b_{31} & \frac{a_{31}}{4a_{32}}(-5a_{31}^2 + 4\sqrt{15}a_{30}a_{32} + 7a_{32}^2 - 5b_{31}^2)
\end{pmatrix}$$

$$Z = \begin{pmatrix}
2a_{32} & 0 & -\sqrt{\frac{5}{2}}a_{31}\\
0 & 2a_{32} & \sqrt{\frac{5}{2}}b_{31}\\
-\sqrt{\frac{5}{2}}a_{31} & \sqrt{\frac{5}{2}}b_{31} & \frac{5a_{31}^2 + 3a_{32}^2 + 5b_{31}^2}{4a_{32}}
\end{pmatrix} \tag{8}$$

With the definition

$$I_1 = -\mathrm{i}\frac{\partial}{\partial\phi_1}, \quad I_2 = -\mathrm{i}\frac{\partial}{\partial\phi_2}, \quad I_3 = -\mathrm{i}\frac{\partial}{\partial\phi_3},$$

we can write down

$$T_{\rm rot} = -\frac{\hbar^2}{2B_3}\frac{1}{3a_{32}^3}Z_{ij}\frac{\partial}{\partial\phi_i}\frac{\partial}{\partial\phi_j} =$$

$$-\frac{\hbar^2}{2B_3}\left[\frac{2}{3a_{32}^2}\left(\frac{\partial^2}{\partial\phi_1^2}+\frac{\partial^2}{\partial\phi_2^2}\right)+\frac{5a_{31}^2+3a_{32}^2+5b_{31}^2}{12a_{32}^4}\frac{\partial^2}{\partial\phi_3^2}-\right.$$

$$\left.\frac{\sqrt{10}a_{31}}{6a_2^3}\left(\frac{\partial}{\partial\phi_1}\frac{\partial}{\partial\phi_3}+\frac{\partial}{\partial\phi_3}\frac{\partial}{\partial\phi_1}\right)-\frac{\sqrt{10}b_{31}}{6a_{32}^3}\left(\frac{\partial}{\partial\phi_2}\frac{\partial}{\partial\phi_3}+\frac{\partial}{\partial\phi_3}\frac{\partial}{\partial\phi_2}\right)\right] =$$

$$\frac{\hbar^2}{2B_3}\left[\frac{2}{3a_{32}^2}I^2+\frac{5(a_{31}^2+b_{31}^2-a_{32}^2)}{12a_{32}^4}I_3^2-\frac{\sqrt{10}}{6a_{32}^3}\left[a_{31}(I_1I_3+I_3I_1)+b_{31}(I_2I_3+I_3I_2)\right]\right]. \tag{10}$$

From above quantized operator of kinetic energy we see that the collective motion of a pure octupole deformed system can be in general treated as vibrations(a_{30}, a_{31}, a_{32} and b_{31} vibrations) in body-fixed frame, and rotation of whole system about the axes of lab-system, as well as the coupling between vibrations and rotation. However, the quantized operator of kinetic energy is very complicated. In fact, in all body-fixed frames defined in Ref. [7], the quantized operators of kinetic energy are all very complicated, which shows that the collective motion in system with general octupole deformation is very complicated. However, for some special octupole deformation, the coupling between vibrations and rotation disappears, the collective motion of octupole deformed system is simple, and we will discuss them in the following paragraph.

3 Collective Spectra Built on Some Special Equilibrium Shapes

To an octupole deformed system with only a_{30} and a_{32} deformations, the collective kinetic energy is then reduced to the form

$$T_3 = \frac{1}{2}B_3(\dot{a}_{30}^2+\dot{a}_{32}^2)+\frac{1}{2}B_3\sum_i\omega_i^2\mathcal{J}_i,$$

where

$$\mathcal{J}_1 = 6a_{30}^2+4a_{32}^2+2\sqrt{15}a_{30}a_{32},$$
$$\mathcal{J}_2 = 6a_{30}^2+4a_{32}^2-2\sqrt{15}a_{30}a_{32}, \tag{1}$$
$$\mathcal{J}_3 = 4a_{32}^2.$$

With the further substitution $a_{30}=\beta\cos\gamma$, $a_{32}=\beta\sin\gamma$, the kinetic energy can be represented as

$$T_3 = \frac{1}{2}B_3(\beta^2+\beta^2\dot{\gamma}^2)+\frac{1}{2}B_3\sum_i\omega_i^2\mathcal{J}_i,$$

where

$$\mathcal{J}_1 = 2\beta^2(3-\sin^2\gamma+\sqrt{15}\sin\gamma\cos\gamma),$$
$$\mathcal{J}_2 = 2\beta^2(3-\sin^2\gamma-\sqrt{15}\sin\gamma\cos\gamma),$$
$$\mathcal{J}_3 = 4\beta^2\sin^2\gamma.$$

The corresponding matrix of metric is

$$G = \begin{pmatrix} 1 & & & & \\ & \beta^2 & & & \\ & & \mathcal{I}_1 & & \\ & & & \mathcal{I}_2 & \\ & & & & \mathcal{I}_3 \end{pmatrix} \tag{15}$$

and the determinant of matrix of metric can be calculated as

$$g = \det G = \beta^2 \mathcal{I}_1 \mathcal{I}_2 \mathcal{I}_3 = 16\beta^8 (9 - 21\sin^2\gamma + 16\sin^4\gamma)\sin^2\gamma. \tag{16}$$

The Pauli prescription replaces the kinetic energy by the operator

$$T_3 = T_{vib} + T_{rot},$$

$$T_{vib} = -\frac{\hbar^2}{2B_3}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin\gamma\sqrt{9-21\sin^2\gamma+16\sin^4\gamma}}\frac{\partial}{\partial\gamma}\sin\gamma\sqrt{9-21\sin^2\gamma+16\sin^4\gamma}\frac{\partial}{\partial\gamma}\right),$$

$$T_{rot} = \frac{\hbar^2}{2B_3\beta^2}\left(\frac{I_1^2}{1+8\sin^2(\gamma+\alpha)} + \frac{I_2^2}{1+8\sin^2(\gamma-\alpha)} + \frac{I_3^2}{4\sin^2\gamma}\right),$$

here $\alpha = \mathrm{tg}^{-1}\sqrt{\frac{5}{3}} \approx 52°$. When the deformation potential energy is involved in Eq. (17), the quantized Hamiltonian built on above special octupole deformations is derived out, which is similar to the Bohr Hamiltonian built on quadruple deformation. When the freedoms of vibration are frozen, the rotational spectra are shown in Fig. 1. From Fig. 1 one sees that when γ goes to zero, the lowest levels of even angular momentum of rotational states approach constant, while the rest levels of rotational states goes to infinity. It means that rotational motion about symmetry axis of a system with odd angular momentum is forbidden in quantum mechanics. When $\gamma = 90° - \alpha$, the rotational levels appear valley in the states 2_2, 3, 4_3, 5_2, 6_4 and so on, which show these states are relatively stable and the octupole deformed nuclei exist the metastable states in the case of $\gamma = 90° - \alpha$. In fact, the shape of this system with $\gamma = 90° - \alpha$ deformation possesses D_{3h} symmetry. With the γ increasing, these rotational levels go up. When $\gamma = \alpha$, these rotational levels go to top and appear peak, which shows the deformation, with $\gamma = \alpha$ is unstable. When $\gamma = 90°$, a_{30} goes to naught and the nuclear surface possesses only a_{32} deformation, the shape of this system possesses T_d symmetry. The rotational Hamiltonian is then reduced to a spherical top, so the rotational levels with same angular momentum are degenerate. These degenerate levels are also lower, which shows that a_{32} is a more important octupole deformation.

4 Conclusions

The collective motion of a pure octupole deformed system has been treated as vibrations (a_{30}, a_{31}, a_{32} and b_{31} vibrations) in body-fixed frame and rotation of whole system about the axes of lab-system, as well as the coupling between vibrations and rotation. The quantized operator of kinetic energy is derived out. The collective spectra built on some special equilibrium shape are obtained with a lot of the interesting properties.

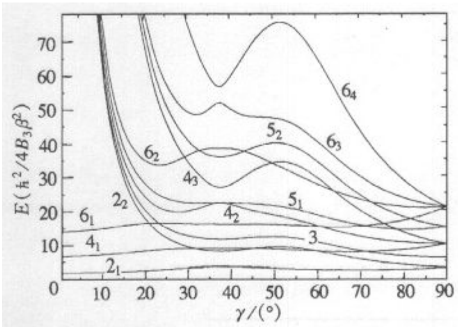

Fig. 1. The rotational spectra built on the special shape with only a_{30} and a_{32} deformations.

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八极形变系统的集体运动*

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摘要 利用一组内部坐标和三个欧拉角描述八极形变系统的集体运动，动能被分解成三部分：体坐标系下的形变振动，体坐标系围绕实验室系的转动以及振动和转动的耦合．量子化的动能算符被导出，一些特殊八极形变下的集体谱被讨论．

关键词 八极形变 集体运动 体坐标框架

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