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Letter

Mixing Effect on the Scalar Glueball Mass in QCD Sum Rules*

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Abstract We consider the mass dependence on the mixing between a pure glueball and a normal $\bar{q}q$ meson in QCD sum rules. In the normal correlation functions, the gluonic current and $\bar{q}q$ current are assumed to couple to both glueball and $\bar{q}q$ states. By using the low-energy theorem, we construct a sum rule for the mixing correlation function (the gluonic current and the $\bar{q}q$ current). Through these relationships and based on the assumption of two states (one glueball and one $\bar{q}q$ meson) dominance, we find the masses of the 0^{++} lowest-lying glueball and $\bar{q}q$ meson.

Key words glueball, sum rules, low-energy theonem, mixing effect

1 Introduction

The existence of bound gluon states, glueballs, is a direct consequence based on the QCD self-interactions among gluons. Although there are several glueball candidates experimen-tally, there is no conclusive evidence on them. People recently pay particular attention to two states: $f_0(1500)(J=0)$[1] and $f_J(1750)(J=0)$[2]. Experiments of BES, MARKIII and Crystal Barrel show they have a large content of gluons, however, neither of them appears to be a pure meson or a pure glueball[3]. Theoretically, the property of glueballs has been investigated in the framework of lattice QCD approach and many phenomenological models inspired by QCD. Even in the lattice gauge calculation, there are different predictions for the 0^{++} glueball[4]. Some years ago, the mass of the 0^{++} glueball was predicted around 700—900 MeV. Recently, IBM group[4] predicts the lightest 0^{++} glueball mass: (1710 ± 63) MeV, and UK QCD group[4] gives the estimated mass: (1625 ± 92) MeV respectively. The

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improvement of determination of the 0^{++} glueball mass originates from the more accuracy of the lattice technique. However, so far the uncertainty of the determination is obvious.

V. A. Novikov et al[5] first tried to estimate the scalar glueball mass by using QCD sum rules[6], but they only took the mass to be 700 MeV by hand because of uncontrolled instanton contributions. Since then, P. Pascual and R. Tarrach[7], S. Narison[8] and J. Bordes et al[9] presented their calculation on the scalar glueball mass in the framework of QCD sum rules. They only considered the perturbative part and the leading condensates in the correlators and got a lower mass prediction around 700—900 MeV. The radiative corrections were took into account by E. Bagan and T. Steele[10]. By choosing appropriate moment, they got the glueball mass prediction around 1.7 GeV since in their calculation the one-loop $\langle \alpha_s G^2 \rangle$ correction played a important role, while two of us found that the radiative corrections are not so important for the determination of the glueball mass[11]. Obviously, there are some uncertainties in the determination of the scalar glueball mass. In this paper, we consider the effect of the mixing between the pure glueball and the normal $\bar{q}q$ mesons. In QCD sum rules, the gluonic currents and quark currents couple both to glueball states and $\bar{q}q$ states so that there are some exotic form factors to be determined. By using the low-energy theorem, we can construct the sum rule for the mixing correlation function (one gluonic current and one quark current). Through these relationship and based on the assumption of two states (lowest-lying states of glueball and $\bar{q}q$ meson) dominance, we find the mass of 0^{++} glueball is around 1.9 GeV, a little higher than the one without taking account of mixing effect, and the mass of 0^{++} $\bar{q}q$ meson is around 1.0 GeV, a little lower than pure $\bar{q}q$ case.

The paper is organized as follows. In Sec. 2 a brief review about the calculation of the mass of physical state from QCD sum rules is given. In Sec. 3 we give the correlators of the scalar currents with $I=0$ from OPE and Low-energy theorem. The spectral density of the scalar current and the numerical results are presented in Sec. 4. The last section is reserved for a summary.

2 QCD Sum Rules

Let us consider the correlator

$$\Pi(q^2) = i\int e^{iqx}\langle 0|T\{j(x),\ j(0)\}|0\rangle dx, \tag{1}$$

where $j(x)$ is the current with definite quantum numbers.

In the deep Euclidean domain $(-q^2 \to \infty)$, it is suitable to carry out operator product expansion (OPE)

$$T\{j(x)j(0)\} = \sum_n C_n(q^2)O_n, \tag{2}$$

where the $C_n(q^2)$ are Wilson coefficients, and the correlator can be expressed in term of the vacuum expectation values of the local operators O_n

On the other hand, the imaginary part of $\Pi(q^2)$ in the Minkovski domain (at positive values of q^2), which is called the spectral density, is relevant with the physical observables. Therefore, we can extract some information of the hadrons from QCD by using the dispersion relation

$$\Pi(q^2) = \frac{(q^2)^n}{\pi}\int \frac{\mathrm{Im}\Pi(s)}{s^n(s-q^2)}\mathrm{d}s + \sum_{k=0}^{n-1} a_k(q^2)^k, \tag{3}$$

where a_k are some subtraction constants originated from the facial divergence of $\Pi(q^2)$. In order to keep control the convergence of the OPE series and enhance the contribution of the lowest lying resonance to the spectral density, we perform the standard Borel transformation on both sides of Eq. (3). However, in practice, it may be more convenient to use the moments R_k which is defined as

$$\begin{aligned} R_k(\tau, s_0) &= \frac{1}{\tau}\hat{L}[(q^2)^k\{\Pi(Q^2) - \Pi(0)\}] - \frac{1}{\pi}\int_{s_0}^{+\infty} s^k \mathrm{e}^{-s\tau}\mathrm{Im}\Pi^{\{pert\}}(s)\mathrm{d}s \\ &= \frac{1}{\pi}\int_0^{s_0} s^k \mathrm{e}^{-s\tau}\mathrm{Im}\Pi(s)\mathrm{d}s, \end{aligned} \tag{4}$$

where $\hat{L}$ is the Borel transformation, τ is the Borel transformation parameter, and s_0 is the starting point of the continuum threshold. We will see the role of R_k in the following analysis.

3 Correlators from QCD approach

In this paper, we focus on the 0^{++} glueball and $\bar{q}q$ meson, so the gauge-invariant scalar gluonic current is chosen as

$$j_1(x) = \alpha_s G^a_{\mu\nu}G^a_{\mu\nu}(x) \tag{5}$$

and the scalar current of quark with isospin $I = 0$ is chosen as

$$j_2(x) = \frac{1}{\sqrt{2}}(\bar{u}u(x) + \bar{d}d(x)). \tag{6}$$

Through operator products expansion, the correlator of $j_1(x)$ without radiative corrections was given by[10]

$$\Pi_1(q^2) = a_0(Q^2)^2\ln(Q^2/\nu^2) + b_0\langle\alpha_s G^2\rangle$$

$$+ c_0 \frac{\langle gG^3 \rangle}{Q^2} + d_0 \frac{\langle \alpha_s^2 G^4 \rangle}{(Q^2)^2} \tag{7}$$

and the correlator of $j_2(x)$ was given by[12]

$$\Pi_2(q^2) = a_0' Q^2 \ln(Q^2/\nu^2) + \frac{3}{Q^2} \langle m\bar{q}q \rangle + \frac{1}{8\pi Q^2} \langle \alpha_s G^2 \rangle + \frac{b_0'}{(Q^2)^2} \langle \bar{q}q \rangle^2, \tag{8}$$

where $Q^2 = -q^2 > 0$, and

$$a_0 = -2\left(\frac{\alpha_s}{\pi}\right)^2 \left(1 + \frac{51}{4}\frac{\alpha_s}{\pi}\right),$$

$$c_0 = 8\alpha_s^2,$$

$$b_0 = 4\alpha_s \left(1 + \frac{49}{12}\frac{\alpha_s}{\pi}\right),$$

$$d_0 = 8\pi\alpha_s,$$

$$a_0' = \frac{3}{8\pi^2}\left(1 + \frac{13\alpha_s}{3\pi}\right),$$

$$b_0' = -\frac{176}{27}\pi\alpha_s,$$

$$\langle \alpha_s G^2 \rangle = \langle \alpha_s G^a_{\mu\nu} G^a_{\mu\nu} \rangle,$$

$$\langle gG^3 \rangle = \langle g f_{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \rangle,$$

$$\langle \alpha_s^2 G^4 \rangle = 14\langle (\alpha_s f_{abc} G^a_{\mu\rho} G^b_{\rho\nu})^2 \rangle - \langle (\alpha_s f_{abc} G^a_{\mu\rho} G^b_{\lambda\nu})^2 \rangle$$

In order to estimate the vacuum expectation values of higher dimension operators, the following equations are obtained by the vacuum saturation approximation[6]

$$\langle \alpha_s^2 G^4 \rangle = \frac{9}{16} \langle \alpha_s G^2 \rangle,$$

$$\langle \bar{q}\sigma_{\mu\nu}\lambda^a q \bar{q}\sigma_{\mu\nu}\lambda^a q \rangle = -\frac{16}{3}\langle \bar{q}q \rangle^2, \tag{9}$$

$$\langle \bar{q}\gamma_\mu \lambda^a q \bar{q}\gamma_\mu \lambda^a q \rangle = -\frac{16}{9}\langle \bar{q}q \rangle^2.$$

By using the Low-energy theorem[13], we can construct another correlator with the quark current and the gluonic current

$$\lim_{q\to 0} i \int \mathrm{d}x e^{iqx} \langle 0 | T\left[\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d),\ \alpha_s G^2\right] | 0 \rangle = \frac{72\sqrt{2}\pi}{29} \langle \bar{q}q \rangle. \tag{10}$$

From (5), (7) and (8), we have

$$
\begin{aligned}
R_0(\tau, s_0) &= -\frac{2a_0}{\tau^3}[1-\rho_2(s_0\tau)] + c_0\langle gG^3\rangle + d_0\langle\alpha_s^2 G^4\rangle\tau,\\
R_1(\tau, s_0) &= -\frac{6a_0}{\tau^4}[1-\rho_3(s_0\tau)] - d_0\langle\alpha_s^2 G^4\rangle,\\
R_2(\tau, s_0) &= -\frac{24a_0}{\tau^5}[1-\rho_4(s_0\tau)],\\
R'_0(\tau, s_0) &= \frac{a'_0}{\tau^2}[1-\rho_1(s_0\tau)] + 3\langle m\bar{q}q\rangle + \frac{1}{8\pi}\langle\alpha_s G^2\rangle + b'_0\tau\langle\bar{q}q\rangle^2,\\
R'_1(\tau, s_0) &= \frac{2a'_0}{\tau^3}[1-\rho_2(s_0\tau)] - b'_0\langle\bar{q}q\rangle^2,
\end{aligned}
\tag{11}
$$

where

$$
\rho_k(x) \equiv e^{-x}\sum_{j=0}^{k}\frac{x^j}{j!}. \tag{12}
$$

R_k and R'_k in (11) are the moments corresponding to $\Pi_1(q^2)$ and $\Pi_2(q^2)$ respectively.

To proceed the numerical calculation, the standard values of the following parameters is chosen

$$
\begin{aligned}
\langle\alpha_s G^2\rangle &= 0.06\mathrm{GeV}^4,\\
\langle gG^3\rangle &= (0.27\mathrm{GeV}^2)\langle\alpha_s G^2\rangle,\\
\langle\bar{q}q\rangle &= -(0.25\mathrm{GeV})^3,\\
\langle m\bar{q}q\rangle &= -(0.1\mathrm{GeV})^4,\\
\Lambda_{\overline{\mathrm{MS}}} &= 200\mathrm{MeV},\\
\alpha_s(m^2_{\mathrm{glueball}}) &= 0.28.
\end{aligned}
$$

4 Spectral density of the scalar currents and numerical results

Now let's proceed to discuss the spectral density. First, we define the couplings of the currents to the physical states as

$$
\begin{aligned}
\langle 0|j_1|Q\rangle &= f_{12}m_2, \quad \langle 0|j_1|G\rangle = f_{11}m_1,\\
\langle 0|j_2|Q\rangle &= f_{22}m_2, \quad \langle 0|j_2|G\rangle = f_{21}m_1,
\end{aligned}
\tag{13}
$$

where m_1 and m_2 refer to the mass of the lowest-lying glueball state $|G\rangle$ and the lowest-lying $\bar{q}q$ state $|Q\rangle$ respectively. If there is no mixing, $f_{12} = f_{21} = 0$. However, in the real world, the physical state is neither pure glueball state nor pure quark state, the mixing effect should not be omitted without any reasonable arguments.

After choosing the two resonances plus continuum states approximation, the spectral density of the currents of $j_1(x)$ and $j_2(x)$ read respectively

$$\mathrm{Im}\Pi_1(s) = m_2^2 f_{12}^2 \delta(s - m_2^2) + m_1^2 f_{11}^2 \delta(s - m_1^2) + \frac{2}{\pi} s^2 \alpha_s^2 \theta(s - s_0), \tag{14}$$

$$\mathrm{Im}\Pi_2(s) = m_2^2 f_{22}^2 \delta(s - m_2^2) + m_1^2 f_{21}^2 \delta(s - m_1^2) + \pi a_0' s \theta(s - s_0). \tag{15}$$

By using Eq. (5), it is not difficult to obtain

$$R_0 = \frac{1}{\pi} \{ m_2^2 e^{-m_2^2 \tau} f_{12}^2 + m_1^2 e^{-m_1^2 \tau} f_{11}^2 \}, \tag{16}$$

$$R_1 = \frac{1}{\pi} \{ m_2^4 e^{-m_2^2 \tau} f_{12}^2 + m_1^4 e^{-m_1^2 \tau} f_{11}^2 \}, \tag{17}$$

$$R_2 = \frac{1}{\pi} \{ m_2^6 e^{-m_2^2 \tau} f_{12}^2 + m_1^6 e^{-m_1^2 \tau} f_{11}^2 \}, \tag{18}$$

$$R_0' = \frac{1}{\pi} \{ m_2^2 e^{-m_2^2 \tau} f_{22}^2 + m_1^2 e^{-m_1^2 \tau} f_{21}^2 \}, \tag{19}$$

$$R_1' = \frac{1}{\pi} \{ m_2^4 e^{-m_2^2 \tau} f_{22}^2 + m_1^4 e^{-m_1^2 \tau} f_{21}^2 \}, \tag{20}$$

In the meanwhile, based on the assumption that the states $|G\rangle$ and $|Q\rangle$ saturate l.h.s of (10), we get

$$\lim_{q \to 0} i \int \mathrm{d}x e^{iqx} \left\langle 0 \middle| T\left[\frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d), \alpha_s G^2 \right] \middle| 0 \right\rangle = f_{22} f_{12} + f_{21} f_{11}. \tag{21}$$

The next step is to equate the QCD side with the hadrons side one by one, then a set of equations about the masses and couplings are obtained. Given various of reasonable parameters s_0 and τ, a series of the two states' masses are extracted through this series of equations. Our result is illustrated in Fig. 1, the solid line corresponds to the glueball and the doted line corresponds to the meson. The points of the plateau compatible to the parameters are regarded as the mass prediction points.

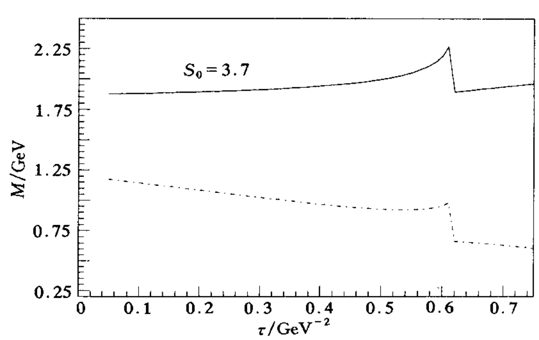

Fig.1 Mass M versus Borel parameter τ

doted line corresponds to the meson. solid line corresponds to the glueball.

It is shown that the mass prediction varies slightly with the s_0 and the $s_0 = 3.7\ \mathrm{GeV}^2$ is the best favorable value for the mass determination, there is no platform for τ above 0.6 GeV^{-2} in this figure. The masses of glueball and quark state read: glueball (m_1) with mass around 1.9 GeV and meson (m_2) with mass around 1.0 GeV. It is found that the glueball mass here is a little higher than the pure glueball state but the quark state mass is a little lower than the pure quark state.

5 Summary

In this paper we study the effect of mixing between lowest-lying 0^{++} glueball and quark state. The physical states as composite resonances, which include both gluon component and quark component, are considered. The spectral density is saturated with two physical resonances, not only the couplings of gluonic current to both glueball state and quark state, but also the couplings of quark current to quark state and glueball state are taken account of through this way. Through the Low-energy theorem and different moments, the masses of glueball and normal meson from a set of coupled equations are predicted: glueball mass is around 1.9 GeV, which is a little higher than the one without mixing (~ 1.7 GeV), while mass of the quark state is around 1.0 GeV. a little lower than the pure quark state (~ 1.1 GeV). We conclude that the glueball mass is not sentitve to the mixing effect.

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混合效应在 QCD 求和规则下对标量胶球质量的影响*

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摘要　在 QCD 求和规则的框架下考虑了纯胶球和普通介子态的混合效应对二者质量的影响．在关联函数的构造中，胶子流和夸克流都被认为既和胶球态有耦合，也和夸克态有耦合．利用 QCD 低能定理，构造了一个混合关联函数．通过这些关系及两共振态近似，得出了 0^{++} 胶球和夸克态的质量，它们与各自的纯态的质量相差不大．

关键词　胶球　求和规则　低能定理　混合效应

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