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Chaotic Motion of Classical Particles in Axially Symmetric Potentials with Octupole Deformation

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The fact that there are more prolate-deformed nuclei than oblate-ones in nature is found to be related to the chaotic motion of nucleons. The primary investigation with classical theory showed that the particle motion in the potential with oblate-plus-octupole deformation could become chaotic at a smaller deformation strength as compared with the case of prolate-plus-octupole deformation. The reason is that the negative curvature in the potential surface could appear at a smaller deformation strength for the former one.

Key words: octupole deformation, potential surface, chaotic motion.

1. INTRODUCTION

In the nuclear independent particle model, any individual nucleon is considered to move in a mean potential field which is produced by all other nucleons. This mean potential field may be spherical, as for double magic nuclei; it may be deformed, as for deformed nuclei. These are fundamental postulations of the shell model which is very successful for describing nuclear structure and nuclear collective states. Deformed nuclei often show a nonlinear character, but to observe and study nuclei from the point of view of the non-linearity is still a new attempt. In recent years, the study of classical chaotic motion for a non-linear Hamiltonian system showed that even very simple systems

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with a few degrees of freedom may show colorful chaotic features [1, 2]. Thus, the investigation of regular and chaotic nucleon motion in the nucleus will possibly shed light on connecting the investigation of nuclear property with the non-linear problem which exists widely in various aspects of the physics world. In this aspect some primary studies showed that when a spherical mean field begins to deform, and if the deformation is described by spherical harmonics with the multipolarity higher than octupole order, then the nuclear motion may change from a regular pattern into a chaotic one at a strong enough deformation strength [3, 4].

Recently W.D. Heiss *et al.* [5] found that in a prolate-plus-octupole deformation potential (P-case), there is no chaotic motion found even if the deformation strength is rather great, and in an oblate-plus-octupole deformation potential (O-case), there is chaotic motion found even if the deformation strength is rather small. The significance of the discovery is not only interesting in the sense of any theoretical study, but also interesting as there are more prolate nuclei than oblate nuclei in nature, and also more prolate triaxial deformed nuclei than oblate triaxial deformed nuclei. Heiss *et al.* have studied the problem both classically and quantum-mechanically. However, they did not survey the origin of the phenomenon. Their current work investigates this origin in the classical way primarily.

2. DYNAMICAL CONDITIONS

2.1. Axially symmetric octupole deformed potential

As in Ref. [5], a single particle is governed by the following potential

$$V(\rho, z) = \frac{m\omega^2}{2}\left[\rho^2 + \frac{z^2}{b^2} + \lambda\frac{2z^3 - 3z\rho^2}{\sqrt{\rho^2 + z^2}}\right], \tag{1}$$

where $\rho^2 = x^2 + y^2$, and x, y, and z are Cartesian coordinates. The first two terms in the square brackets stand for the axially symmetric harmonic oscillator potential, and the frequency in the z direction is different from those in x, y directions, $\omega_x = \omega_y = b\omega_z$. For $\lambda = 0$ the parameter b measures the quadrupole deformation which is prolate for $b > 1$ and oblate for $b < 1$. The third term in the square brackets is the axially symmetric octupole deformed potential, which is proportional to r^2Y_{30}. Where Y_{30} is the octupole spherical harmonics, $r^2 = \rho^2 + z^2$. For any given quadrupole deformation, either for $b > 1$ or $b < 1$, there is a critical value λ_c so that for $|\lambda| > \lambda_c$ the potential no longer binds a particle. For prolate potentials, $b = 2 > 1$ is chosen, $\lambda_c = \frac{1}{2b^2}$; for oblate ones with $b = 0.5$ the critical value is $\lambda_c = 1.64$.

2.2. Dynamical equations

In the potential by Eq. (1) the canonical equations of a classical particle are

$$\begin{cases} \dfrac{\partial z}{\partial t} = mP_z, \qquad \dfrac{\partial \rho}{\partial t} = mP_\rho, \\ \dfrac{\partial P_z}{\partial t} = \dfrac{-m\omega^2}{2}\left\{\dfrac{2z}{b^2} + \lambda\dfrac{3(2z^2 - \rho^2)}{(\rho^2 + z^2)^{\frac{1}{2}}} - \lambda\dfrac{z^2(2z^2 - 3\rho^2)}{(\rho^2 + z^2)^{\frac{3}{2}}}\right\}, \\ \dfrac{\partial P_\rho}{\partial t} = \dfrac{-m\omega^2}{2}\left\{2\rho - \lambda\dfrac{6z\rho}{(\rho^2 + z^2)^{\frac{1}{2}}} - \lambda\dfrac{\rho z(2z^2 - 3\rho^2)}{(\rho^2 + z^2)^{\frac{3}{2}}}\right\}, \end{cases} \tag{2}$$

which are non-linear equations, bearing no analytical solutions. So they have to be solved numerically. Here the Runge-Kutta algorithm is used, and the time step is taken as 0.1 fm.

3. NEGATIVE CURVATURE OF THE POTENTIAL SURFACE AND THE PARTICLE CHAOTIC MOTION

In a Hamiltonian system $H(q,p) = \frac{p^2}{2} + V(q)$, the phase difference between two arbitrary neighboring trajectories $\{q_1(t), p_1(t)\}$ and $\{q_2(t), p_2(t)\}$ is $\xi(t) = q_1(t) - q_2(t)$, $\eta(t) = p_1(t) - p_2(t)$. The analysis of the trajectory stability [3, 6] showed that if the difference changes with time exponentially then for a system with two degrees of freedom there must exist

$$C = \frac{\partial^2 V}{\partial q_1^2}\frac{\partial^2 V}{\partial q_2^2} - \left(\frac{\partial^2 V}{\partial q_1 \partial q_2}\right)^2 < 0, \tag{3}$$

where the derivatives are along the fiducial trajectory q_1, and scalars q_1, q_2 are the components of the two dimensional vector q.

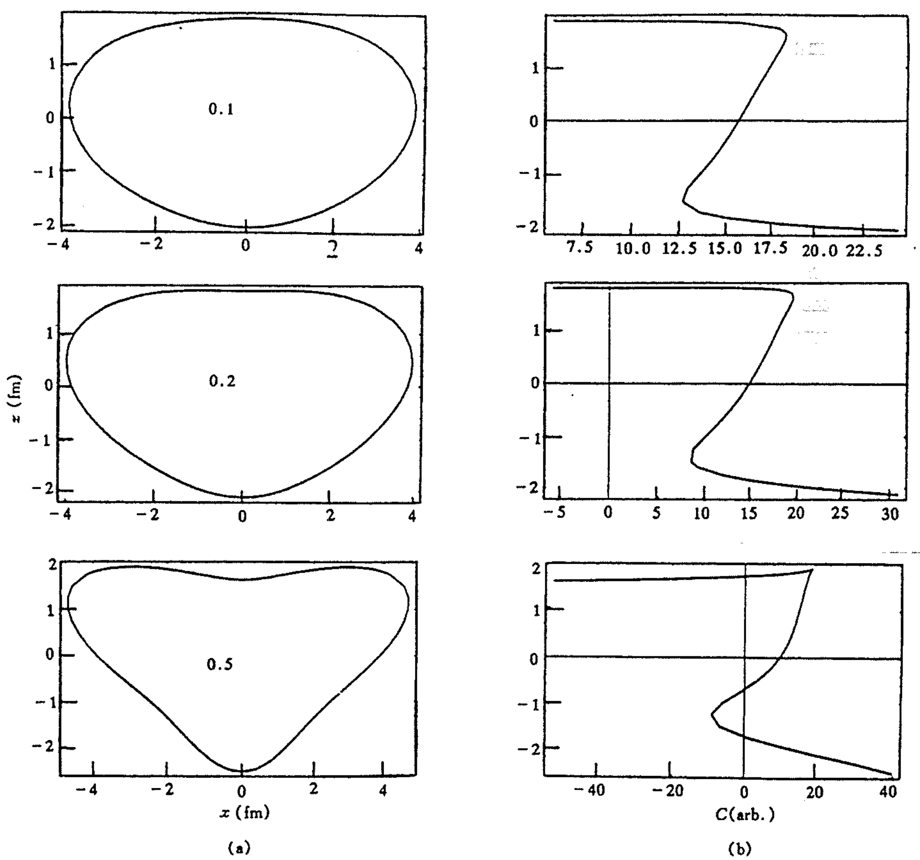

Fig. 1
Equipotential lines in the $x - z$ plane at $y = 0$ for oblate ($b = 0.5$)-plus-octupole deformation. (b) The curvature of the corresponding equipotential lines.

It turns out that the sign of C is the same as the sign of the Gaussian curvature of the potential surface, which is [7]:

$$K(q_1,q_2) = \frac{\dfrac{\partial^2 V}{\partial q_1^2}\dfrac{\partial^2 V}{\partial q_2^2} - \left(\dfrac{\partial^2 V}{\partial q_1 \partial q_2}\right)^2}{\left[1 + \left(\dfrac{\partial V}{\partial q_1}\right)^2 + \left(\dfrac{\partial V}{\partial q_2}\right)^2\right]^2} \tag{4}$$

So it can be considered that the negative curvature of the potential surface would cause the exponential separation of the neighboring trajectories. If a particle sweeps the potential area with negative curvature for many times, the phase difference of the neighboring trajectories would greatly change and this would lead the particle motion to become chaotic. For the sake of convenience, here the curvature of the potential surface is defined by C (Eq. (3)).

In Fig. 1 the projections of the potential surface $V(\rho, z)$ onto the $x - z$ plane at $y = 0$ are plotted on the left column. From the top to the bottom λ/λ_c are 0.1, 0.2, and 0.5, respectively, with $\lambda_c = 1.64$. Since the potential is scale invariant as $V(\gamma r) = \gamma^2 V(r)$, potential surfaces with different energy are all similar, and only one equipotential line is needed to visualize its form. In the right column the

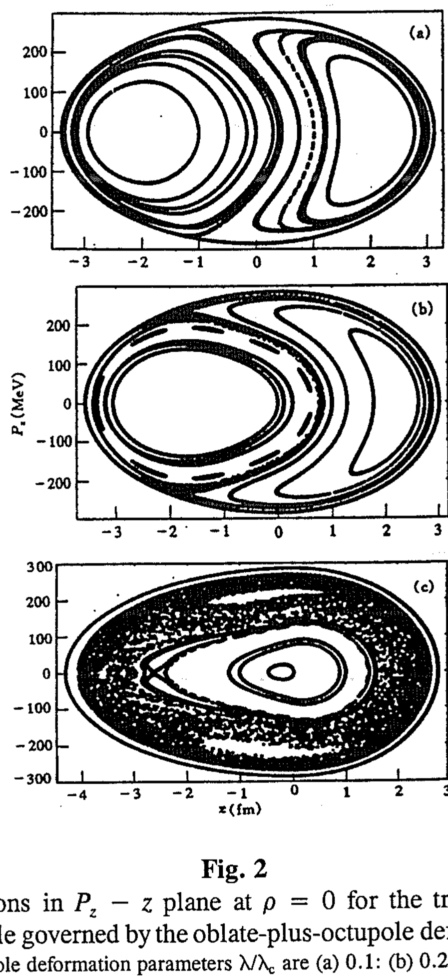

Fig. 2

Poincaré sections in $P_z - z$ plane at $\rho = 0$ for the trajectories of the classical particle governed by the oblate-plus-octupole deformed potential. The octupole deformation parameters λ/λ_c are (a) 0.1: (b) 0.2; and (c) 0.5.

curvature of the corresponding potential surface is shown. For $\lambda = 0.1\lambda_c$ there is no negative curvature in the potential surface. For $\lambda = 0.2\lambda_c$ at the top of the potential surface there is a small region with $C < 0$. For $\lambda = 0.5\lambda_c$ the negative curvature region in the top of the potential grows and two other regions with negative curvature appear on the both sides of the potential.

In Fig. 2 the corresponding Poincaré sections in the $P_z - z$ plane at $\rho = 0$ are shown. It is indicated in the figure that for (a) there is no negative curvature in the potential surface and a thoroughly regular Poincaré section is observed. For (b) corresponding to a small region with negative curvature the irregular scattering of points is distributed in a very thin ring in the Poincaré section. For (c) corresponding to the three regions of stronger negative curvature (see Fig. 1) in the potential surface, a large area of the irregular region can be observed in the Poincaré section.

For the P-case the corresponding calculations were completed taking $b = 2$. In the left column of Fig. 3 the equipotential lines in the $x - z$ plane at $y = 0$ are shown for $\lambda/\lambda_c = 0.5$, 0.6, and 0.7, respectively, with $\lambda_c = \frac{1}{2b^2}$. For $\lambda/\lambda_c \geq 0.6$ there are small regions of negative curvature in both sides of the potential surface, and the curvatures are shown on the right column. Note that the scale of z corresponding to the x coordinate was strongly compressed so that the prolates appear to be not very long. Otherwise if the scales in z and x are taken to be the same, the equipotential surfaces will be greatly elongated. Correspondingly, in Fig. 4 the Poincaré section indicated that for (a), $\lambda/\lambda_c = 0.5$, there is no negative curvature in the potential surface, and the Poincaré section shows a regular pattern; for (b), $\lambda/\lambda_c = 0.6$ and for (c) $\lambda/\lambda_c = 0.7$, corresponding to the observed growing negative

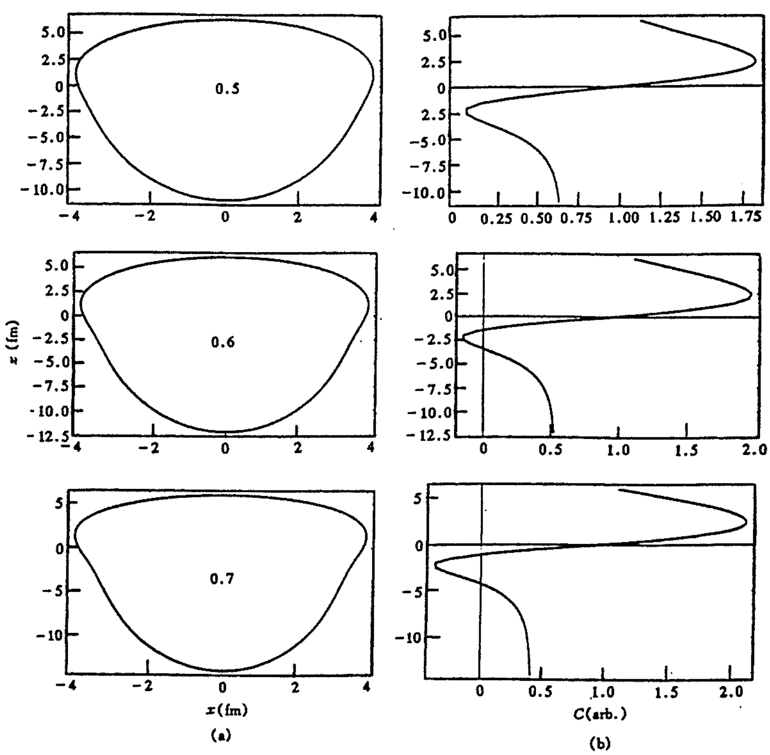

Fig. 3
(a) Equipotential lines in the $x - z$ plane at $y = 0$ for prolate ($b = 2$)-plus-octupole deformation. (b) The curvature of the corresponding equipotential lines.

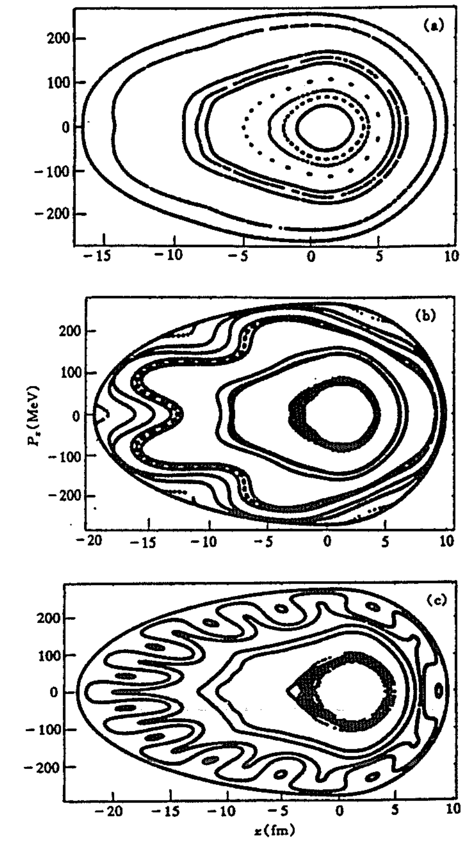

Fig. 4

Poincaré sections in $P_z - z$ plane at $\rho = 0$ for the trajectories of the classical particle governed by the prolate-plus-octupole deformed potential.

The octupole deformation parameters λ/λ_c are (a) 0.5; (b) 0.6; and (c) 0.7.

curvature in the potential surface, there are growing chaotic areas in the Poincaré sections. In Heiss'work, for the P-case ($b = 2$) with $\lambda/\lambda_c = 0.67$, there is no chaotic structure observed in their $P_z - z$ Poincaré section at $\rho = 0$. The reason is the following: In the same potential, trajectories with different initial conditions appear in certain regions of the potential, respectively. In the case that one did not find out the right initial condition which leads to the chaotic trajectory, the chaotic trajectory would be lost and the corresponding chaotic region in the Poincaré section would be missed. Actually, the chaotic region we have discovered was located in the blank region of their $P_z - z$ Poincaré section. For O-case ($b = 0.58$) with $\lambda/\lambda_c = 1/3$ they obtained a chaotic, almost structureless Poincaré section, which coincides with our results.

4. SUMMARY

For a regular integrable system, there are as many integrals of motion as degrees of freedom. Confined by the integrals of motion, each degree of freedom can only distribute in a finite area of the phase space instead of an arbitrary infinite distribution. If in the case that one integral of motion was destroyed, the corresponding degree of freedom would be free from the restriction and could possibly extend infinitely. However, the KAM (Kolmogrov-Arnold-Moser) theorem attests that this is not the case. For a non-integrable system which is sufficiently close to a integrable one, if the non-integrability can be treated as a perturbance to the integrable Hamiltonian function, then under a small perturbation (nearly integrable system), the pattern of the motion could be still consistent with that of the integrable system. The torus formed by trajectories in the phase space still exists, but is deformed; in the case that the perturbance is strong enough (strong non-integrable system), the pattern of motion will change qualitatively into chaotic and the torus will be destroyed. The KAM theorem is thus considered as, "the greatest breakthrough in the developing history of Newtonian mechanics" [8].

To make a motion chaotic, there must exist a turbulence which destroys the integrability of the system. At least one integral of motion has to be destroyed, and the strength of the turbulence must be strong enough to accomplish the destruction. Here the demand for this so-called "strong enough" power is a qualitative one and it is not indicated by the KAM theorem. In this work, the incorporation of the octupole deformation term destroyed the angular momentum conservation of the system, and only the projection of the angular momentum onto z axis is the integral of motion. For smaller deformation strength λ, when there is no negative curvature in the octupole deformed potential surface, there is no chaotic motion. As the deformation grows, the negative curvature appears in the nuclear potential surface, and there will always be a chaotic trajectory found as long as one searches carefully for it in the phase space. So, as a supplement to the KAM theorem, we have provided the turbulence strength required for chaotic motion. The requirement of the turbulence strength to cause chaotic motion is that there must be a negative curvature in the deformed potential surface. The zero curvature will be a critical point for the motion changing from regular to chaotic. It has been mentioned in the text that the negative curvature would appear at bigger λ/λ_c for the P-case than that in the O-case. As it was shown in Fig. 3 for the P-case the chaotic motion would appear at very strong deformation, and usually the nuclear deformation cannot reach that extent. Thus, for the P-case, there is a remarkably stronger stability against chaos than for the O-case. Furthermore, when the axial symmetry is broken, and the deformation turns towards a three axial deformation, for example, the deformation could be expressed by Y_{3m}, where $m \neq 0$, though the corresponding integrals of motion were broken, the nucleon motion could be still regular as long as the curvature was positive everywhere in the deformed potential surface. Then the corresponding trajectories were still confined in a torus, but it is deformed. To sum up, the negative curvature in the potential surface appears at smaller deformation for the O-case than for the P-case so that the chaotic motion is usually observed in the O-case. The corresponding nuclei become more unstable when some of their integrals of motion are destroyed. The reflection of the influence of the negative curvature in quantum mechanics will be an interesting and compelling problem.

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