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Nuclear Structure of Nuclei in Lead Region (II) Non-Unique First-Forbidden β Decays of Nuclei ^{208}Tl, ^{208}Pb, and $^{206-208}$Hg

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The non-unique first-forbidden β decays of ^{208}Tl, ^{208}Pb, and $^{206-208}$Hg are calculated in terms of the shell model with different interactions and model space. The calculated log f_0t value very sensitively depends on the effective interactions used in diagonalizing the energy matrix. The β decay modes for ^{206}Hg and ^{208}Hg are also compared.

Key words: nuclear structure, non-unique first forbidden β decay, shell model.

1. INTRODUCTION

In Ref. [1], we calculated the spectra and wave-functions of ^{208}Tl, etc. The results showed that the spectra obtained with different interactions are very similar, but the wave-functions are different.

The β decay of nucleus can provide more information about the nuclear structure and test more sensitively the interaction used to diagonalize the Hamiltonian. For light and mid-heavy nuclei, the β decay modes, in general, are allowed the Fermi type and Gamow-Teller transition since the protons and neutrons fill the same shell. On the contrary, for heavy nuclei, only forbidden β decay can happen because the proton and neutron fill different shells. For the nuclei near ^{208}Pb, the first-forbidden β decay are the most important one. In this paper, we discuss the properties of the first-forbidden β decays of ^{208}Tl, ^{208}Pb, and $^{206-208}$Hg.

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Due to the Pauli blocking, the β decays of ^{208}Tl, ^{207}Hg, and ^{208}Hg can convert a neutron in 126–184 shells into a $\pi h_{11/2}$ or decay into the p-h excitation states of their daughter nuclei. It is clear that the ground-state of ^{208}Tl decays predominantly into the 5^- and 4^- states of ^{208}Pb via first-forbidden β transition [2]. The ground-state of ^{207}Hg decays into the $7/2^-$, $9/2^-$, and $11/2^-$ states of ^{207}Tl, although the assignments of these states are still uncertain [3] experimentally. For the β decay of ^{208}Hg, it is totally unclear except a measured half-life (42 min) provided by Zhang Li *et al.* [4]. According to the similar shell structure of ^{208}Hg to ^{208}Tl and ^{207}Hg, one can expect that the ground-state of ^{208}Hg(0^+) decays predominantly into the 0^-, 1^-, and 2^- states of ^{208}Tl via the first-forbidden β transition. It has been seen that this negative state has relatively high excited energies, and the p-h excitation is important for these states. The neglected single-particle orbits are also important for these states. These lead to the difficulties in determining accurately the wave-functions of these states and analyzing the β decays of ^{207}Hg and ^{208}Hg. We now calculate $\log f_0 t$ for ^{208}Tl, ^{208}Pb, and $^{206-208}$Hg by using the wave-functions obtained in Ref. [1].

2. MATRIX ELEMENTS AND TRANSITION RATES OF THE FIRST-FORBIDDEN B DECAY

In the impulse approximation, the first-forbidden β operator can be classified into two types [5]: the first type includes four operators r and $[r, \sigma]^R$ ($R = 0, 1, 2$ is the order) which arise from the expansion of the lepton wave-function and are nonrelativistic; the second type includes operators γ_5 and α which are caused by the mixture of the big and small components in the weak current and are relativistic. These operators [6] are listed in Table 1.

There exists a relation between M^x and M^y due to the current conservation [7],

$$\langle J_f T_f \| \alpha\tau \| J_i T_i \rangle = E_\gamma \langle J_f T_f \| i r C_1 \tau \| J_i T_i \rangle, \tag{1}$$

where $C_L = \left[\dfrac{4\pi}{2L+1}\right]^{1/2} Y_L$, $|J_i T_i\rangle$ $|J_f T_f\rangle$ are the initial and final wave-functions, respectively, τ is the isospin operator and E_γ is the energy difference between the final state and the state which is the isobaric analog state to the parent state. For the nuclei with A = 205–212, E_γ can be well expressed as,

$$E_\gamma = \frac{1.412}{0.511}\frac{(Z_i + Z_f)}{2A^{1/3}} - 0.811 - 0.786 + Q(\beta^-), \tag{2}$$

where $Q(\beta^-)$ is the decay energy. Thus, there are five independent β matrix elements: two rank-zero (RO:M_0^S, M_0^T), one rank-one ($R1$: M_1^x, M_1^u), and one rank-two ($R2$: M_2^z).

In the shell model the β matrix elements can be calculated in the following way

$$M_R^a = \sum_{j_i j_f} M_R^a(j_i j_f) = \sum_{j_i j_f} D_R(j_i j_f) M_R^a(j_i j_f, \text{eff}) = \sum_{j_i j_f} D_R(j_i j_f) M_R^a(j_i j_f) q_a(j_i j_f), \tag{3}$$

In this equation, $D_R(j_i j_f)$ are the one-body transition density which can be calculated in the shell model; $M_R^a(j_i j_f)$ is the single-particle matrix element for the transition $j_i \to j_f$ in the impulse approximation; and the quenching factor $q_a(j_i j_f)$ makes the finite size model space and nuclear medium effect correction to $M_R^\alpha(j_i j_f)$ with $R = 0, 1, 2$; $a = T, S, x, u, z$. The β matrix elements are calculated by using oscillator single-particle wave-functions with $\hbar\omega = 41.464A^{-1/3} - 25.0A^{-2/3}$. The average values for the quenching factors q_a in the lead region are q_T~1.15, q_S~0.85, q_u~0.45, and q_x~0.60 [8].

In order to obtain the relation between the transition rate and matrix elements, the lepton wave-functions are expanded into series with respect to αZ, Wr_u, pr_u, and qr_u by using the Behren-Buhring

Table 1
The matrix elements and their symbols for the first-forbidden β decays.

Symbols	Cartesian coordinate	Spherical coordinate	Order
M_0^T	$-C_A \int \gamma_5$	$C_A(4\pi)^{1/2}\langle\gamma_{50}\rangle$	0
M_0^S	$C_A \int i\sigma \cdot r$	$-C_A(4\pi)^{1/2}\langle ir\sigma \cdot T_0^1\rangle$	0
M_1^y	$-C_v \int \alpha$	$C_v(4\pi)^{1/2}\langle \alpha \cdot T_1^0\rangle$	1
M_0^x	$-C_v \int ir$	$-C_v(4/3\pi)^{1/2}\langle irY_1\rangle$	1
M_1^u	$-C_A \int \sigma\Lambda r$	$-C_v(8/3\pi)^{1/2}\langle ir\sigma \cdot T_1^1\rangle$	1
M_2^z	$C_A \int iB_{ij}$	$-C_v(16/3\pi)^{1/2}\langle ir\sigma \cdot T_2^1\rangle$	2

method [9]. Here, α is the fine structure constant, W and p are the energy and momentum of the electron, respectively, q denotes the momentum of the neutrinos, and r_u represents the radius of a uniform charge distribution. In this expansion, the first kind of matrix elements are $M_0^{S'}$, $M_1^{x'}$, and $M_1^{u'}$. Let $r_w' = M_0^{s'}/M_0^S, r_x' = M_1^{x'}/M_1^x$, and $r_u' = M_1^{u'}/M_1^u$. For the nuclei with A–208, $r_w \approx r_x \approx r_u =$ 0.70.

For the non-unique first-forbidden β decay, one can define the average shape factor as:

$$\overline{C(W)} = 9195 \times 10^5 / f_0 t = B_1^{(0)} + B_1^{(1)}, \tag{4}$$

where $B_1^{(0)}$ and $B_1^{(0)}$ are the contributions of the rank-zero (R0) and rank-one (R1) components, respectively, and can be formulated in terms of β matrix elements in the ξ approximation as

$$\begin{aligned} B_1^{(0)} &= [M_1^{(0)}]^2 = [\varepsilon_{\text{mec}} M_0^T + a_S M_0^S]^2, \\ B_1^{(1)} &= [M_1^{(1)}]^2 = [a_u M_1^u - a_x M_1^x]^2. \end{aligned} \tag{5}$$

Here ε_{mec} is the meson-exchange-current (mec) enhancement factor and its value can be chosen as 2.01 $\pm$ 0.05 in our calculation [7]. a_S, a_u, and a_x are defined as

$$\begin{aligned} a_S &= r_w'\xi + \frac{1}{3} W_0, \\ a_u &= r_u'\xi - \frac{1}{3} W_0, \\ a_x &= E_\gamma - r_x'\xi - \frac{1}{3} W_0, \\ \xi &= \frac{\alpha Z}{2r_u}, \end{aligned} \tag{6}$$

where W_0 is the maximum β decay energy.

3. RESULTS

3.1 ^{208}Tl(β^-)^{208}Pb

The ^{208}Tl ground-state decays predominantly into the 5^- and 4^- states of ^{208}Pb. The corresponding branch ratios are 48.7% 5_1^-, 24.5% 5_2^-, and 21.8% 4_1^- [3], respectively. These experimental data are listed in Table 2. The 5^- and 4^- states of ^{208}Pb are calculated with both PKH and SDI.

Table 2

The $\log f_0 t$ values of the first forbidden β decay for nuclei ^{208}Tl.

Transitions	E_x(MeV)	$\log f_0 t$(exp)	$\log f_0 t$(th)
$5^+ \to 5_1^-$	3.198	5.61	6.0
$5^+ \to 5_2^-$	3.708	5.37	5.37
$5^+ \to 4_1^-$	3.475	5.69	5.65

Table 3(a)

The experimental $\log f_0 t$ values of the non-unique first-forbidden β decays for ^{207}Hg.

Transitions	E_x(MeV)	$\log f_0 t$(exp)	$I\beta$(%)
$9/2^+ \to 11/2^-$	1.348	5.0	2
$9/2^+ \to 7,\ 9/2^-$	2.911	6.2	14
$9/2^+ \to 7,\ 9/2^-$	2.985	5.8	32
$9/2^+ \to 7/2^-$	3.104	5.9	16
$9/2^+ \to 7,\ 9,\ 11/2^-$	3.143	6.3	7
$9/2^+ \to 7/2$	3.272	6.5	3
$9/2^+ \to 9,\ 11/2^-$	3.295	6.2	5
$9/2^+ \to 11/2^-$	3.334	6.2	5
$9/2^+ \to 7,\ 9,\ 11/2^-$	3.339	6.3	4

Table 3(b)

The calculated $\log f_0 t$ values of the non-unique first-forbidden β decays for ^{207}Hg.

Transitions	E_x(MeV)	$\log f_0 t$(th)
$9/2^+ \to 11/2^-$	1.435	7.871
$9/2^+ \to 11/2^-$	3.480	5.876
$9/2^+ \to 7/2^-$	3.493	6.868
$9/2^+ \to 7/2^-$	3.584	6.085
$9/2^+ \to 9/2^-$	3.079	6.506
$9/2^+ \to 9/2^-$	3.355	5.376
$9/2^+ \to 9/2^-$	3.644	5.842

The wave-functions for 5_1^-, 5_2^- and 4_1^- can be written as $56\%\nu\,|\,3p_{1/2}^{-1}2g_{9/2};\ 5_1^-\rangle + 26.3\%\pi\,|\,2s_{1/2}^{-1}1h_{9/2};\ 5_1^-\rangle$, $38.7\%\nu\,|\,3p_{1/2}^{-1}2g_{9/2};\ 5_2^-\rangle + 55.5\%\pi\,|\,3s_{1/2}^{-1}1h_{9/2};\ 5_2^-\rangle$, and $95\%\nu\,|\,3p_{1/2}^{-1}2g_{9/2};\ 4_1^-\rangle$, respectively. The transitions $5^+ \to 5_{1,2}^-$ are predominated by $\nu 3p_{1/2} \to \pi 3s_{1/2}$ and $\nu 2g_{9/2} \to \pi 1h_{9/2}$, respectively; and the transition $5^+ \to 4_1^-$ is predominated by $\nu 3p_{1/2} \to \pi 3s_{1/2}$. The calculated $\log f_0 t$ value for 5_1^-, 5_2^-, and 4_1^- are 6.0, 5.37, and 5.65, respectively. These results agree with the data $5.61(5_1^-)$, (5.375_2^-), and $5.69(4_1^-)$ quiet well. The SDI interaction gives the $\log f_0 t$ values of 5_1^- 6.48, which is larger than those results of PKH and the experimental data.

3.2. ^{207}Hg(β^-)^{207}Tl

The experimental [3] and calculated $\log f_0 t$ values for the decay of the ground-state of ^{207}Hg($9/2^+$) into the $11/2^-$, $9/2^-$, and $7/2^-$ states of ^{207}Tl are listed in Tables 3(a) and 3(b), respectively. The calculated ground-state wave-function of ^{207}Hg is dominated by $|\pi 3s_{1/2}^{-2}\nu 2g_{9/2};9/2^+\rangle$ (70%),

Table 4
The calculated $\log f_0 t$ values of the first-forbidden β decays for ^{208}Hg.

Transitions	fE_x(MeV)	$\log f_0 t$(th)
$0^+\to0^-$	2.480	5.374
$0^+\to0^-$	2.945	5.904
$0^+\to1^-$	2.355	6.928
$0^+\to1^-$	2.870	6.281

$|\pi d^{-2}_{3/2}\nu 2g_{9/2};9/2^+\rangle$ (15%), and $|\pi 1h^{-2}_{11/2}\nu 2g_{9/2};9/2^+\rangle$ (5%). The last component $|\pi 1h^{-2}_{11/2}\nu 2g_{9/2};9/2^+\rangle$ determines the transition strength to the first $11/2^-$ state of ^{207}Tl, which is predominated by the $\nu 2g_{9/2}\to\pi 1h_{11/2}$ transition. This transition rate is quite small. The observed branch ratio is 2%. The calculated result of this transition with PKH is $\log f_0 t = 7.87$, which agrees with the experimental value $\log f_0 t = 8.0$ quiet well. However, the SDI interaction gives a smaller value $\log f_0 t = 7.14$ for this transition. It shows again the defect of SDI in describing the β decays of theses nuclei.

The main part of the decay of ^{207}Hg(98%) feeds the transitions of the ground-state of ^{207}Hg into $1p-1h$ excited states (including proton $1p-2h$ components and neutron $2p-1_h$ components). The calculated $\log f_0 t$ values for these transitions shown in Table 3(b), in general, agree with the experimental data very well.

3.3. ^{206}Hg(β^-)^{206}Tl and ^{208}Hg(β^-)^{208}Tl

Both ^{206}Hg and ^{208}Hg are even-even nuclei. It is useful to compare the decays of these two nuclei. The ground state of ^{206}Hg mainly decays into the ground-states 0^- and the second low-lying state 1^- of ^{206}Tl via the first-forbidden decay of $\nu 3p_{1/2}\to\pi 3s_{1/2}$ or $\nu 3p_{1/2}\to\pi 2d_{3/2}$. The calculated $\log f_0 t$ values for these two transitions are 5.20($0^+\to0^-$) and 5.09($0^+\to1^-$), respectively. They agree with the experimental values of 5.42 and 5.23 very well.

^{208}Hg decays into the relative higher excited 0^-, 1^-, and 2^- states of ^{208}Tl, whose wave-functions have very complex structures. The calculated $\log f_0 t$ values for the decays of ^{208}Hg into the 0^- and 1^- states of ^{208}Tl are given in Table 4. These $\log f_0 t$ values are calculated by adopting an estimated decay energy Q-3.1 MeV [11]. The $\log f_0 t$ value of the first-forbidden decay is not sensitive to the decay energy in the ξ approximation. The ground-state wave-function of ^{208}Hg is dominated by configurations $\pi 3s^{-2}_{1/2}\nu 2g^2_{9/2}$(46.77%), $\pi 3s^{-2}_{1/2}\nu 1i^2_{11/2}$(17.77%), and $\pi 2d^{-2}_{3/2}\nu 2g^2_{9/2}$(10.95%). The configuration $\pi 1h^{-2}_{11/2}\nu 2g^2_{9/2}$ only feeds 3.6%. This small configuration gives a small transition rate of $\nu 2g_{9/2}\to\pi 1h_{11/2}$, similar to the result of ^{207}Hg decaying into the $11/2^-$ state of ^{207}Tl. The first 1^- state of ^{208}Tl is dominated by the configuration $\pi 1h^{-1}_{11/2}\nu 2g_{9/2}$(90%). The calculated $\log f_0 t$ value for the transitions ($0^+\to1^-_1$) is 6.93. The second 1^- state of ^{208}Tl is dominated by the neutron $1p-1h$ excited state, and the 0^+ state of ^{208}Hg decays into this state via the $\nu 3p_{1/2}\to\pi 3s_{1/2}$ or $\nu 3p_{1/2}\to\pi 2d_{3/2}$ process, and the calculated value of $\log f_0 t$ is 5.28, which is much smaller than the first one. The first 0^- state of ^{208}Tl is also dominated by the neutron $1p-1h$ excited state. The calculated value of $\log f_0 t$ is 5.37. The second 0^- state is dominated by the proton $2p-1h$ state. The decay mode is $\nu 2g_{9/2}\to\pi 1h_{11/2}$, and $\log f_0 t = 5.90$.

The calculated value of $\log f_0 t$ for the decays of ^{208}Hg are similar to those of ^{206}Hg, because both decays are mainly predominated by the $\nu 3p_{1/2}\to\pi 3s_{1/2}$ or $\nu 3p_{1/2}\to\pi 2d_{3/2}$ transitions. As mentioned above ^{206}Hg decays into the ground-state or the low-lying excited states of ^{206}Tl, while ^{208}Hg decays into high-lying $1p-1h$ excited states. If the provided decay energy of ^{208}Hg is smaller than that of ^{206}Hg, one can expect that the half-life of ^{208}Hg is longer than that of ^{206}Hg. Unfortunately, the Q value for the β decay of ^{208}Hg is unknown in the experiment.

4. CONCLUSIONS

The first-forbidden β decays of ^{208}Tl, ^{208}Pb, and $^{206\text{-}208}$Hg are calculated. With the interaction PKH the calculated results of ^{208}Tl, ^{208}Pb, and ^{206,207}Hg agree with the experimental data. The shows that the PKH interaction is more suitable to describe the effects of the $1p-1h$ excitation. The $\log f_0 t$ values for the decays of the ground-state of ^{208}Hg into the 0^- and 1^- state of ^{208}Tl are also calculated using an estimated Q value. If the provided β decay energy of ^{208}Hg is smaller than that of ^{206}Hg, the half-life of ^{208}Hg would be longer than that of ^{206}Hg.

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