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Skyrme-Hartree-Fock Calculation on Exotic Nuclei

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The ground state properties of the nuclei near $Z = 8$ and $Z = 50$ are investigated by the Skyrme-Hartree-Fock approach with new force parameters SKI2, SKI3, and SKI4. Calculations show that the Skyrme-Hartree-Fock theory with the above force parameters provides a good description of those isotopes. Of the three sets of force parameters, SKI4, which has taken into consideration the isospin-dependence of the spin-orbit force, is the best one. SKI4 is also successful in reproducing the charged isotopes shifts near $Z = 50$.

Key words: Skyrme-Hartree-Fock theory, charged isotopes shifts, spin-orbit force, isospin-dependence, binding energy.

1. INTRODUCTION

For more than two decades, the mean-field theory based on the nucleon-nucleon interactions has enjoyed enormous success in providing a microscopical description of the ground state properties of nuclei [1-5]. The Skyrme force, among various forces, is used widely and can describe the ground state properties of nuclei. The results, including binding energies, various radii (charge radii, proton radii, neutron radii, and matter radii) and surface thickness, calculated by the Skyrme force agree with the experiments. However, it can only used to investigate the nuclei close to the β-stable line. With the improvement of experimental conditions and especially the development of large laser separators, a new field in nuclear physics, i.e., the study of nuclei far from the β-stable line, has opened up and become a hot topic in theoretical nuclear physics.

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However, it was pointed out recently [3-5] that the Skyrme-Hartree-Fock (SHF) theory with the standard parameterization fails to reproduce the observed charged isotopes shifts in Ca, Sr, and Pb elements because these parameters have been obtained by fitting the nuclei near the β-stable line. In order to solve this problem, Reinhard and Flocard [6] have proposed five sets of new Skyrme force parameters in which some nuclei far from the β-stable line have also been included in the fitting process. They have analyzed the problem in detail with these new parameters and found that it is related to the characteristics of the spin-orbit interactions in the standard parameterizations. Among five sets of new force parameters, SKI1, SKI2, and SKI5 have similar spin-orbit interactions to the standard parameterizations; SKI4, which has a generalized spin-orbit interaction, can succeed in reproducing the charged isotopes shifts in Ca, Pb, and Sr elements [6]; SKI3, which has a similar spin-orbit interaction to that in relativistic mean field (RMF) theory but is different from that in SKI2 and SKI4, can reproduce the charged isotopes shifts for Pb isotopes but cannot reproduce the charged isotopes shifts of the Ca and Sr isotopes.

In this paper, the ground state properties of light nuclei will be investigated with the new force parameters and the validity of new force parameters will be checked. Furthermore, the charged isotopes shifts in some nuclei, which have never been calculated satisfactorily, will be reproduced and the properties of some exotic nuclei will also be predicted. Because of the similar spin-orbit term in SKI1, SKI2, and SKI5, only the best one (SKI2) will be used.

2. SKYRME-HARTREE-FOCK THEORY

As the Skyrme-Hartree-Fock (SHF) theory is a standard theory [1-6], here only a short description on the framework is given.

The form of the two-body Skyrme-type interaction can be written as [6]

$$\begin{aligned} V_{12}=&t_0(1+x_0P_\sigma)\delta(r_{12})+\frac{1}{2}t_1(1+x_1P_\sigma)(K'^2\delta(r_{12})+\delta(r_{12})K^2)\\ &+\frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha((r_1+r_2)/2)\,\delta(r_{12})+t_2(1+x_2P_\sigma)(K'\cdot\delta(r_{12})K)\\ &+\mathrm{i}\,W_0(1+x_4P_\tau)(\sigma_1+\sigma_2)(K'\times\sigma(r_{12})K)\,, \end{aligned} \quad (1)$$

where K denotes the operator $(\nabla_1-\nabla_2)/2\mathrm{i}$ acting on the right and K' is the operator $-(\nabla_1-\nabla_2)/2\mathrm{i}$ acting on the left. P_σ and P_τ are the spin-exchange operator and the isospin-exchange operator, respectively. The first two terms in Eq. (1) correspond to the S-wave interaction and the last two terms correspond to the P-wave one, the third one is the three-body force term and this force is equivalent to a two-body density-dependent interaction under the time-reversal invariance [6].

In the standard HF method, the nuclear ground state is represented by a Slater determinant of single-particle states

$$\psi(x_1, x_2\cdots, x_A)=\frac{1}{\sqrt{A!}}\det|\varphi_i(x_j)|\,, \quad (2)$$

where φ denotes the single-particle wave function and in the spherical coordinates it can be written as follows:

$$\varphi_\beta=\frac{R_\beta(r)}{r}Y_{j\beta l\beta m\beta}(\theta,\varphi)\,. \quad (3)$$

The total energy is:

$$\begin{aligned} E=&\langle\psi|(T+V)|\varphi\rangle\\ =&\sum_i\langle i|\frac{p^2}{2m}|i\rangle+\frac{1}{2}\sum_{ij}\langle ij|\bar{v}_{12}|ij\rangle\,, \end{aligned} \quad (4)$$

where $|i\rangle$ and $|ij\rangle$ denote single-particle states and the notation v denotes an antisymmetrized matrix element. The Hartree-Fock equations for Skyrme's interaction can be obtained by variation of the total energy with respect to the single-particle state φ_i [1].

$$\left[-\nabla\cdot\frac{\hbar^2}{2m_q^*(r)}\nabla+U_q(r)+W_q(r)(-\mathrm{i})(\nabla\times\vec{\sigma})\right]\varphi_i=e\varphi_i\ , \tag{5}$$

where

$$\frac{\hbar^2}{2m_q^*(r)}=\frac{\hbar^2}{2m}+\frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right)+t_2\left(1+\frac{1}{2}x_2\right)\right]\rho+\frac{1}{4}\left[t_2\left(\frac{1}{2}+x_2\right)-t_1\left(\frac{1}{2}+x_1\right)\right]\rho_q\ , \tag{6}$$

$$\begin{aligned}U_q={}&t_0\left[\left(1+\frac{1}{2}x_0\right)\rho-\left(x_0+\frac{1}{2}\right)\rho_q\right]+\frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right)+t_2\left(1+\frac{1}{2}x_2\right)\right]\tau\\&+\frac{1}{4}\left[t_2\left(\frac{1}{2}+x_2\right)-t_1\left(\frac{1}{2}+x_1\right)\right]\tau_q+\frac{1}{8}\left[3t_1\left(\frac{1}{2}+x_1\right)+t_2\left(\frac{1}{2}+x_2\right)\right]\nabla^2\rho_q\\&-\frac{1}{8}\left[3t_1\left(1+\frac{1}{2}x_1\right)-t_2\left(1+\frac{1}{2}x_2\right)\right]\nabla^2\rho+\frac{1}{6}t_3\rho^\alpha\left[\left(1+\frac{1}{2}x_3\right)\rho-\left(x_3+\frac{1}{2}\right)\rho_q\right]\\&+\frac{1}{12}\alpha t_3\rho^{\alpha-1}\left[\left(1+\frac{1}{2}x_3\right)\rho^2-\left(x_3+\frac{1}{2}\right)\left(\rho_n^2+\rho_p^2\right)\right],\end{aligned} \tag{7}$$

$$W_q=\frac{1}{2}W_1(\nabla\rho+\nabla\rho_q)+\frac{1}{8}(t_1-t_2)J_q-\frac{1}{8}(t_1x_1+t_2x_2)J\ , \tag{8}$$

here $W_1=W_0(1+x_4)$.

$$\rho=\rho_p+\rho_n\ ,\ \tau=\tau_p+\tau_n\ ,\ \nabla J=\nabla J_p+\nabla J_n$$

ρ, τ, and J denote the nucleon densities, the kinetic energy densities, and the spin densities, respectively. The densities in the spherical representation are [1-5]

$$\rho_q=\sum_i|\varphi_i(r)|^2=\sum_\beta\omega_\beta\frac{2j_\beta+1}{4\pi}\left(\frac{R_\beta}{r}\right)^2, \tag{9}$$

$$\tau_q=\sum_i|\nabla\varphi_i(r)|^2=\sum_\beta\omega_\beta\frac{2j_\beta+1}{4\pi}\left[\left(\partial r\frac{R_\beta}{r}\right)^2+\frac{l_\beta(l_\beta+1)}{r^2}\left(\frac{R_\beta}{r}\right)^2\right], \tag{10}$$

$$\begin{aligned}J_q&=-\mathrm{i}\sum_i\varphi_i^*(r)(\nabla\varphi_i(r)\times\vec{\sigma})\\&=\sum_\beta\omega_\beta\frac{2j_\beta+1}{4\pi}\left[j_\beta(j_\beta+1)-l_\beta(l_\beta+1)-\frac{3}{4}\right]\frac{2}{r}\left(\frac{R_\beta}{r}\right)^2,\end{aligned} \tag{11}$$

where ω_β is the occupation weight of single-particle levels.

Table 1
Skyrme-type parameters of SKI2, SKI3, and SKI4.

	SKI2	SKI3	SKI4
t_0	−1915.43	−1762.88	−1855.83
t_1	438.449	561.608	473.829
t_2	305.446	−227.09	1006.86
t_3	10548.9	8106.2	9703.61
x_0	−0.2108	0.3083	0.4051
x_1	−1.7376	−1.1722	−2.8891
x_2	−1.5336	−1.0907	−1.3252
x_3	−0.1780	1.2926	1.1452
b_4	60.301	94.254	183.097
b'_4	60.301	0.0000	−180.0351

In the conventional Skyrme form the energy function contains a Hartree (symmetry term) and a Fock (antisymmetry term) contribution. If the exchange (Fock-) term for the spin-orbit potential in the last term of Eq. (1) and t_1, t_2 contribution are neglected, one finds [1]

$$W_q(r)=b_4\nabla\rho+b'_4\nabla\rho_q, \tag{12}$$

here $b_4 = W_0(1 + x_4) / 2$, $b'_4 = -W_0 x_4 / 2$. The isospin degrees of freedom appear in Eq. (12) compared with Eq. (8).

The mean field localizes the nucleus and thus breaks transformational invariance which results in an oscillation of the center-of-mass of the nucleus in the mean field. However, the total momentum of the exact nuclear ground state should be zero. A simple and reliable treatment on the center-of-mass correction is [4]:

$$E_{cm}=\frac{\langle P^2_{cm}\rangle}{2Am},$$

$$\langle P^2_{cm}\rangle=\sum_\beta \omega_\beta\langle\varphi_\beta|\hat{p}^2|\varphi_\beta\rangle -\sum_{\alpha\beta}\left(\omega_\alpha\omega_\beta+\sqrt{\omega_\alpha(1-\omega_\alpha)\omega_\beta(1-\omega_\beta)}\right)|\langle\varphi_\alpha|\hat{p}|\varphi_\beta|^2. \tag{13}$$

3. NUMERICAL RESULTS AND DISCUSSION

The Skyrme force parameters SKI2, SKI3, and SKI4 with different spin-orbit terms are used to calculate the ground state properties of nuclei N, O, and F. The values of three sets of force parameters are listed in Table 1. The numerical results for the binding energies, matter radii, neutron radii, and proton radii are listed in Tables 2 and 3, respectively. A self-consistent BCS treatment of the pairing correlations with a strength of G_p = 17 MeV/u for protons and G_n = 23 MeV/u for neutrons is used in the mean field model. The superscripts p and n denote protons and neutrons, respectively. In order to obtain the reliable results, each interaction includes a wave function interaction and a pairing interaction in the program. The angular momenta and parities of the nuclei

Table 2
Binding energies of N, O, and F isotopes.

	Exp. [8]	SKI2	SKI3	SKI4	FIS
^{11}N	58.350	61.85	59.16	58.82	61.4
^{13}N	94.105	95.19	92.93	94.43	93.9
^{15}N	115.492	117.81	116.82	117.27	114.4
^{17}N	123.865	128.37	125.86	125.87	125.8
^{19}N	132.018	138.20	134.23	133.19	134.6
^{21}N	138.79	145.93	140.79	137.65	141.4
^{23}N	142.43(a)	152.69	146.58	141.23	145.4
^{12}O	58.530	63.62	61.33	59.84	61.2
^{14}O	98.733	101.19	99.65	100.39	98.4
^{16}O	127.620	128.61	128.33	128.14	124.2
^{18}O	139.807	142.78	141.21	141.21	139.7
^{20}O	151.371	155.70	152.19	151.92	152.3
^{22}O	162.030	166.54	162.87	160.78	162.4
^{24}O	168.480	175.42	170.80	166.22	168.8
^{26}O	168.430(a)	179.71	174.81	168.74	168.7
^{28}O		183.35	178.66	170.62	167.3
^{17}F	128.22	130.70	129.86	129.92	127.7
^{19}F	147.80	148.08	146.15	147.13	146.9
^{21}F	162.50	164.03	161.10	162.42	162.8
^{23}F	175.21	177.91	174.41	173.66	175.9
^{25}F	183.48	188.91	184.58	181.94	184.6
^{27}F	185.83	195.14	190.78	186.71	187.0
^{29}F	186.73(a)	200.85	196.98	191.03	187.8

The value labeled with (a) is the datum estimated from systematic trends.

containing a value nucleon (or one hole) outside the closed shell depend only on the occupation of the last nucleon (or hole). So the effects of proton pairing forces for N and F nuclei are not taken into account.

The experimental binding energies from Ref. [8] are listed in the first column and the numerical results are listed in the second to the fourth columns for three sets of parameters in Table 2, respectively. The results for nuclei near the β-stable line agree well with the experimental data for SKI2, SKI3, and SKI4. With the increase in the number of neutrons, a smaller difference from the different sets of parameters appears and the difference increases with the neutron excess. The results from SKI2 are similar to that from normal Skyrme parameters, and the deviation between numerical results and experimental data increases for neutron-rich nuclei. For SKI4, the calculated values agree well with the experimental value and the relative error is within 1%. The results of SKI3 are between

Table 3
Various radii of N, O, and F isotopes.

	SKI2				SKI3				SKI4			
	r_m	r_c	r_n	r_p	r_m	r_c	r_n	r_p	r_m	r_c	r_n	r_p
^{11}N	2.60	2.83	2.29	2.77	2.62	2.82	2.34	2.77	2.63	2.87	2.35	2.78
^{13}N	2.63	2.69	2.58	2.65	2.62	2.69	2.56	2.66	2.62	2.68	2.58	2.65
^{15}N	2.72	2.70	2.74	2.70	2.69	2.69	2.70	2.67	2.70	2.72	2.71	2.70
^{17}N	2.79	2.68	2.87	2.67	2.78	2.68	2.87	2.65	2.81	2.68	2.90	2.67
^{19}N	2.90	2.68	3.03	2.66	2.90	2.67	3.05	2.63	2.95	2.68	3.12	2.64
^{21}N	2.99	2.68	3.14	2.65	2.98	2.67	3.14	2.62	3.06	2.67	3.26	2.63
^{23}N	3.10	2.68	3.28	2.64	3.09	2.67	3.28	2.62	3.17	2.67	3.38	2.62
^{12}O	2.73	2.94	2.38	2.88	2.71	2.91	2.40	2.86	2.76	2.98	2.45	2.89
^{14}O	2.70	2.78	2.61	2.76	2.67	2.76	2.58	2.73	2.69	2.76	2.63	2.73
^{16}O	2.74	2.77	2.74	2.75	2.71	2.75	2.70	2.71	2.73	2.75	2.72	2.74
^{18}O	2.79	2.75	2.84	2.72	2.77	2.73	2.83	2.69	2.79	2.73	2.85	2.70
^{20}O	2.86	2.74	2.96	2.70	2.83	2.72	2.93	2.67	2.89	2.72	3.02	2.67
^{22}O	2.93	2.73	3.07	2.68	2.91	2.72	3.06	2.66	3.01	2.73	3.20	2.66
^{24}O	3.05	2.73	3.22	2.67	3.04	2.72	3.22	2.66	3.11	2.74	3.32	2.65
^{26}O	3.20	2.79	3.39	2.70	3.17	2.78	3.37	2.69	3.23	2.78	3.45	2.68
^{28}O	3.30	2.84	3.51	2.73	3.26	2.83	3.46	2.72	3.32	2.84	3.54	2.72
^{17}F	2.77	2.88	2.72	2.81	2.75	2.87	2.69	2.80	2.76	2.88	2.70	2.82
^{19}F	2.79	2.84	2.81	2.77	2.78	2.84	2.80	2.75	2.78	2.83	2.81	2.75
^{21}F	2.84	2.82	2.93	2.74	2.84	2.82	2.91	2.73	2.87	2.81	2.97	2.72
^{23}F	2.90	2.81	3.01	2.72	2.90	2.81	3.01	2.72	2.94	2.81	2.09	2.70
^{25}F	3.02	2.82	3.18	2.72	3.02	2.82	3.18	2.72	3.13	2.85	3.31	2.77
^{27}F	3.16	2.87	3.34	2.75	3.14	2.87	3.32	2.76	3.17	2.87	3.63	2.74
^{29}F	3.26	2.93	3.45	2.97	3.23	2.92	3.41	2.80	3.28	2.94	3.45	2.87

Charge radii r_c, proton radii r_p, neutron radii r_n, and matter radii r_m.

that of SKI2 and SKI4. There are different kinds of spin-orbit potentials in SKI2, SKI3, and SKI4. SKI2 has the normal Skyrme force whose neutron and proton spin-orbit potential is proportional, respectively, to $\nabla(2\rho_n + \rho_p)$ and $\nabla(2\rho_p + \rho_n)$. In SKI3, $b'_4 = 0$, so both the neutron and proton spin-orbit potentials are proportional to $\Delta\rho$. The structure of the spin-orbit potential is similar to that in the relativistic mean field theory. SKI4 has a different spin-orbit potential from SKI2 and SKI3, where the neutron spin-orbit potential is proportional to $\nabla\rho_p$ but the proton's potential is proportional to $\nabla\rho_n$. So it has an isospin-dependent spin-orbit interaction. It is obvious that the inclusion of the

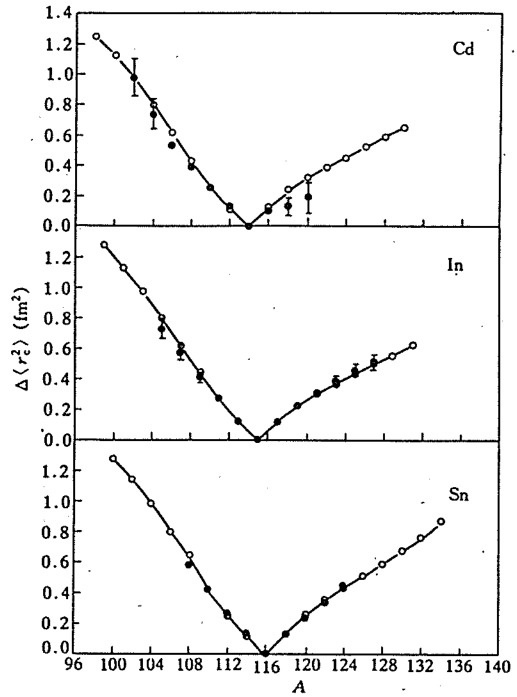

Fig. 1

Charged isotopes shifts for Cd, In, and Sn nuclei calculated with force parameters SKI4. ● and ○ denote experimental data and numerical results, respectively. The experimental isotope shifts for Cd [11,13], In [13], and Sn [12] nuclei are also shown.

isospin dependence in the spin-orbit force by SKI4 leads to a better agreement with experimental data. Lombard's numerical results (FIS) [7] are listed in the last column in Table 2 and those results are not as accurate as SKI4. Our results show that the binding energy of ^{26}O is higher than that of ^{24}O and it means that ^{26}O are stable for the two-neutron emission. We predict a similar behavior for ^{23}N, ^{28}O, and ^{29}F which is in accordance with that obtained in the relativistic mean-field theory.

The results of various radii of N, O, and F nuclei are listed in Table 3, there is a similar trend as the binding energy in Table 2. The results of SKI2, SKI3, and SKI4 have similar radii for the nuclei near the β-stable line. The neutron radii from SKI4 are obviously greater than that from SKI2 and SKI3 as the increase of neutron numbers. It comes from two causes: one is that the neutron single particle level gets shallow; the other is that the spin-orbit split gets large. The causes are related to the isospin-dependent spin-orbit interaction. Some neutron-rich nuclei with halo structure, which have abnormally large neutron and matter radii, have been observed in experiments, but there is not any satisfactory theoretical explanation. The large neutron radii in SKI4 indicate that the modification of the isospin-dependent spin-orbit interaction may be a possible way to explain that kind of exotic nuclei. There are not enough data for the experimental radii, especially for nuclei far from the β-stable line. The empirical values of the various radii obtained from the experiments for N, O, and F nuclei are [7,10]: $r_c(^{15}\mathrm{N}) = 2.61$ fm, $r_m(^{17}\mathrm{N}) = (2.80 \pm 0.04)$ fm, $r_n(^{17}\mathrm{N}) = (2.94 \pm 0.15)$ fm, $r_m(^{19}\mathrm{N}) = (2.79 \pm 0.06)$ fm, $r_n(^{19}\mathrm{N}) = (2.88 \pm 0.09)$ fm, $r_m(^{16}\mathrm{O}) = (2.63 \pm 0.06)$ fm, $r_c(^{16}\mathrm{O}) = 2.70$ fm, $r_n(^{16}\mathrm{O}) = (2.59 \pm 0.11)$ fm, $r_c(^{18}\mathrm{O}) = 2.75$ fm, $r_m(^{20}\mathrm{O}) = (3.00 \pm 0.35)$ fm, $r_n(^{20}\mathrm{O}) = (3.20 \pm 0.59)$ fm, and $r_c(^{19}\mathrm{F}) = 2.90$ fm. Those experimental data agree well with our numerical results.

Table 4
Binding energies of nuclei near $Z = 50$.

	Exp. [8]	SKI4		Exp. [8]	SKI4		Exp. [8]	SKI4
^{98}Cd	820.89 (a)	825.19	^{99}In	821.64 (a)	826.63	^{100}Sn	824.48 (a)	830.76
^{100}Cd	843.84 (a)	845.07	^{101}In	845.28 (a)	847.36	^{102}Sn	848.91 (a)	852.52
^{102}Cd	865.140	863.08	^{103}In	867.611	866.29	^{104}Sn	871.850	872.46
^{104}Cd	885.840	880.86	^{105}In	888.635	885.03	^{106}Sn	893.870	892.25
^{106}Cd	905.140	898.76	^{107}In	908.858	903.86	^{108}Sn	914.598	915.01
^{108}Cd	923.403	916.86	^{109}In	927.924	922.85	^{110}Sn	934.562	932.34
^{110}Cd	940.642	934.98	^{111}In	945.969	941.85	^{112}Sn	953.528	952.22
^{112}Cd	957.016	952.70	^{113}In	963.090	960.13	^{114}Sn	971.570	971.47
^{114}Cd	972.599	971.87	^{115}In	979.403	976.60	^{116}Sn	988.679	992.02
^{116}Cd	987.440	985.77	^{117}In	994.951	991.75	^{118}Sn	1004.95	1008.0
^{118}Cd	1001.57	996.51	^{119}In	1009.85	1006.3	^{120}Sn	1020.54	1020.8
^{120}Cd	1014.98	1009.3	^{121}In	1024.13	1019.9	^{122}Sn	1035.52	1035.5
^{122}Cd	1027.72	1021.3	^{123}In	1037.86	1032.9	^{124}Sn	1049.96	1049.6
^{124}Cd	1040.00	1033.0	^{125}In	1051.06	1045.4	^{126}Sn	1063.88	1063.2
^{126}Cd	1051.76	1044.5	^{127}In	1063.72	1057.8	^{128}Sn	1077.34	1076.6
^{128}Cd	1062.87	1055.7	^{129}In	1075.84	1069.8	^{130}Sn	1090.39	1089.8
^{130}Cd		1065.3	^{131}In	1087.210	1080.2	^{132}Sn	1102.91	1101.3

The value labeled with (a) is the datum estimated from systematic trends.

In order to study the effect of the isospin-dependent spin-orbit force, we calculate the charged isotopes shifts of Cd, In, and Sn nuclei near $Z = 50$ with SKI4. In Fig. 1, we plot the charged isotopes shifts $r_c^2(A) - r_c^2(\mathrm{ref})$ for those nuclei calculated with respect to a reference nucleus in each chain; here $r_c^2(\mathrm{ref})$ denotes the charge radii of the reference nucleus. The nuclei with $N = 66$, ^{114}Cd, ^{115}In, and ^{116}Sn are reference nuclei in our calculation. Why do we choose those nuclei as the reference nuclei? There are two reasons: one is that the experimental data can compare to the numerical results, the other is that $N = 64$ is a subshell and there is a large charged isotopes shift near the closed shell or subshell. It is seen from the figure that the calculation is successful in reproducing the charged isotopes shifts. Some nuclei far from the β-stable line, which have no experimental data up to now, are also shown in the figure. The neutron and proton spin-orbit strengths do not depend on the isospin in the conventional Skyrme parameters, and this makes the differences of the nuclear densities for nuclei near the β-stable line and the nuclei far from the β-stable line very small. The spin-orbit potential of SKI4 is obviously different from the conventional Skyrme parameters, the neutron spin-orbit strength depends on $\nabla\rho_p$ but the proton's strength depends on $\nabla\rho_n$. This suggests that the correlations between neutrons and protons are strengthened due to the spin-orbit interactions. As the neutron number is near the magic number or semi-magic number, the correlations between protons and neutrons vary remarkably and a large influence on the charged isotopes shifts appears. This may be the reason why there is a large value in charged isotopes shifts as the neutron number is near the closed shell or subshell. The results of binding energies of three isotope nuclei near $Z = 50$ are listed in Table 4 and the relative deviation between numerical results and experimental data is within 1%. It also indicates that the isospin-dependent spin-orbit interactions can not only describe the properties of the nuclei near the β-stable line but also the nuclei far from the β-stable line.

In a word, the SHF results with new force parameters agree well with the experimental data on the binding energies and the various radii. The agreement is reasonably good for SKI4 which has the isospin-dependent spin-orbit interactions. With SKI4, not only can the ground state properties of light nuclei N, O, and F be described very well but also the charged isotopes shifts of nuclei near $Z = 50$ can be well reproduced. Our investigation indicates a possible way for the study of the exotic nuclei far from the β-stable line.

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