

# Multichannel Coupling in the Hadronic Decay Process $J / \psi \rightarrow \omega \pi^+ \pi^-$

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The coupling problem of the three different intermediate processes (e.g.,  $J / \psi \rightarrow \omega f_2$  (1270),  $f_2 \rightarrow \pi^+ \pi^-$  and  $J / \psi \rightarrow b_1^\pm$  (1235)  $\pi^\mp$ ,  $b_1^\pm \rightarrow \omega \pi^\pm$ ) included in the hadronic decay process  $J / \psi \rightarrow \omega \pi^+ \pi^-$  is discussed. The consideration of the coupling effect is very important for measuring the parameters of the resonances  $f_2$  and  $b_1^\pm$  and the helicity amplitude ratios of these reactions precisely.

**Key words:** multichannel coupling, helicity, hadronic decay process.

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## 1. INTRODUCTION

The hadronic decay process  $J / \psi \rightarrow \omega \pi^+ \pi^-$  is observed in the  $5\pi$  meson final state. Its branching ratio is  $Br(J / \psi \rightarrow \omega \pi^+ \pi^-) = (7.2 \pm 1.0) \times 10^{-3}$  [1]. Except the contribution from the reaction  $J / \psi \rightarrow \omega f_2$  (1270),  $f_2 \rightarrow \pi^+ \pi^-$  the contribution from  $J / \psi \rightarrow b_1^\pm$  (1235)  $\pi^\mp$ ,  $b_1^\pm \rightarrow \omega \pi^\pm$  is also included for the  $\omega \pi^+ \pi^-$  channel. Their branching ratios are, respectively,

$$\begin{aligned} Br(J / \psi \rightarrow \omega f_2 (1270)) &= (4.3 \pm 0.6) \times 10^{-3}, \\ Br(J / \psi \rightarrow b_1^\pm (1235) \pi^\mp) &= (3.0 \pm 0.5) \times 10^{-3}. \end{aligned} \quad (1)$$

The branching ratio of  $f_2 \rightarrow \pi^+ \pi^-$  is about 57% and the dominant decay channel of  $b_1^\pm$  is  $\omega \pi^\pm$  [1].

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The  $\omega\pi^+\pi^-$  channel can be also observed through  $J/\psi \rightarrow \omega f_0$  (980),  $f_0 \rightarrow \pi^+\pi^-$ . This channel has been observed by DM2 collaboration [2]. But it was not observed by MarkII and MarkIII collaborations [3]. The result from BES supports the result from DM2 [4]. Its branching ratio is  $Br(J/\psi \rightarrow \omega f_0(975)) = (1.4 \pm 0.5) \times 10^{-4}$ ,  $Br[f_0(975) \rightarrow \pi^+\pi^-] \approx 50\%$ . Because the branching ratio of this process is smaller, when we discuss the decay channels  $\omega f_2$  and  $b_1^\pm \pi^\mp$  we can neglect the influence of the decay channel  $\omega f_0$  (980). But this decay channel is very important for determining the properties of  $f_0$  (980), so when we study the  $f_0$  (980) we must consider the effects of the above three decay channels.

We must consider the influence of the  $b_1^- \pi^+$  decay channel when we select  $\omega f_2$  (1270) in  $\omega\pi^+\pi^-$  final state and vice versa. It represents that there are two overlapping regions in the Dalitz plot of the event points of the three reaction channels. If we do the angular distribution fit or the moment analysis the coupling terms will appear, and the multichannel coupling effect will occur. Because of the symmetry we will only discuss the coupling between  $J/\psi \rightarrow \omega f_2$ ,  $f_2 \rightarrow \pi^+\pi^-$ , and  $J/\psi \rightarrow b_1^- \pi^+$ ,  $b_1^- \rightarrow \omega\pi^-$  in one overlapping region. In Ref. [5] the values of the helicity amplitude ratios  $x$ ,  $y$ ,  $z_1$ ,  $z_2$  for the process  $J/\psi \rightarrow \omega f_2$  (1270),  $f_2 \rightarrow \pi^+\pi^-$  were given. It is important for investigating the dynamical mechanism of the reaction. However, the above coupling effect cannot be neglected if we want to improve the measurement precision. Because there are a large number of hadronic resonance states in a 1.0-2.5 GeV energy region this kind of coupling problem is of universal significance. We must pay attention to it and conduct the study and analysis carefully.

## 2. ANGULAR DISTRIBUTION FORMALISM FOR REACTION $e^+e^- \rightarrow J/\psi \rightarrow \omega f_2, f_2 \rightarrow \pi^+\pi^-$ , AND $e^+e^- \rightarrow J/\psi \rightarrow b_1^- \pi^+, b_1^- \rightarrow \omega\pi^-$ WITHOUT COUPLING

The helicity formalism of angular distribution for the reaction,

$$e^+e^- \rightarrow J/\psi \rightarrow \omega f_2, f_2 \rightarrow \pi^+\pi^- \quad (2)$$

has been discussed [6]. Considering the parity conservation and time-reversal invariance we have

$$\begin{aligned} W_2(\theta_f, \theta_1^*, \varphi_1^*) &\propto (1 + \cos^2\theta_f) [(3 \cos^2\theta_1^* - 1)^2 + \frac{3}{2} y^2 \sin^4\theta_1^* + \frac{3}{2} z_2^2 \sin^2 2\theta_1^*] \\ &- \sin 2\theta_f [\sqrt{3x} (3 \cos^2\theta_1^* - 1) - \frac{3}{\sqrt{2}} xy \sin^2\theta_1^* \\ &- \sqrt{3} z_1 z_2 (3 \cos^2\theta_1^* - 1)] \sin 2\theta_1^* \cos \varphi_1^* \\ &+ \sin^2\theta_f \{ [\sqrt{6} y \sin^2\theta_1^* (3 \cos^2\theta_1^* - 1) - \frac{3}{2} z_2^2 \sin^2 2\theta_1^*] \cos 2\varphi_1^* \\ &+ 3x^2 \sin^2 2\theta_1^* + z_1^2 (3 \cos^2\theta_1^* - 1)^2 \}. \end{aligned} \quad (3)$$

where  $\theta_f$  is the angle between the  $f_2$  (1270) meson momentum  $p_f$  and the  $e^+$  momentum  $p_+$  in the  $J/\psi$  rest frame  $K$ ; the  $z_1$  axis is chosen to be along the direction of the momentum  $p_f$ ,  $e^+e^-$  beams lie in the  $x_1-z_1$  plane, the direction of the  $y_1$  axis is taken as the direction of  $(p_+ \times p_-)$ ;  $(x_1, y, z_1) \equiv K_1$  frame.  $(\theta_1^*, \varphi_1^*)$  describe the direction of the momentum of  $\pi^+$  meson in the center of mass system  $K_1^*$  of  $f_2$  meson. Where  $x, y, z_1, z_2$  are four independent helicity amplitude ratios of the process (2)

$$x = \frac{A_{1,1}}{A_{0,1}}, \quad y = \frac{A_{2,1}}{A_{0,1}}, \quad z_1 = \frac{A_{0,0}}{A_{0,1}}, \quad z_2 = \frac{A_{1,0}}{A_{0,1}}. \quad (4)$$

where  $A_{\lambda_f, \lambda_\omega}$  is the helicity amplitude, and  $\lambda_f$  and  $\lambda_\omega$  are helicities of  $f_2$  meson and  $\omega$  meson, respectively.

Let us consider the reaction

$$e^+e^- \rightarrow J/\psi \rightarrow b_1^- \pi^+, b_1^- \rightarrow \omega \pi^-, \tag{5}$$

Its angular distribution is

$$\begin{aligned} W_1(\theta_b, \varphi_b, \theta_2^*, \varphi_2^*) &\propto \sum_{\lambda_b, \lambda_\omega, \lambda_b^*, \lambda_\omega^*} \delta_{\lambda_b, \pm 1} \cdot B_{\lambda_b, 0} B_{\lambda_\omega^*, 0} D_{\lambda_b^*, \lambda_\omega}^{1*}(\varphi_b, \theta_b, -\varphi_b) D_{\lambda_b, \lambda_\omega^*}^1(\varphi_b, \theta_b, -\varphi_b) \\ &\times D_{\lambda_b, \lambda_\omega}^{1*}(\varphi_2^*, \theta_2^*, -\varphi_2^*) D_{\lambda_b^*, \lambda_\omega^*}^1(\varphi_2^*, \theta_2^*, -\varphi_2^*) |C_{\lambda_b, 0}|^2 \\ &\propto 2(1 + \cos^2 \theta_b) [(1 + \cos^2 \theta_2^*) + \xi^2 \sin^2 \theta_2^*] \\ &+ 2 \sin^2 \theta_b [2 \eta^2 (\sin^2 \theta_2^* + \xi^2 \cos^2 \theta_2^*) + (1 - \xi^2) \sin^2 \theta_2^* \cos 2(\varphi_b - \varphi_2^*)] \\ &- 2 \sin 2 \theta_b \cdot \eta (1 - \xi^2) \sin 2 \theta_2^* \cos(\varphi_b - \varphi_2^*). \end{aligned} \tag{6}$$

where  $(\theta_b, \varphi_b)$  describe the direction of the  $b_1^-$  meson momentum  $p_b$  in the  $J/\psi$  rest frame  $K$ ; the  $z_2$  axis is chosen to be along the direction of the momentum  $p_b$ , the  $e^+e^-$  beams are in the  $x_2-z_2$  plane, the  $y_2$  axis is taken to be along the direction of  $(p_+ \times p_-)$ , and  $(x_2, y_2, z_2)$  are the  $K_2$  frame.  $(\theta_2^*, \varphi_2^*)$  describe the direction of  $\omega$  meson momentum in the center of mass system  $K_2^*$  of  $b_1^-$ . In view of the parity conservation and time-reversal invariance there are two helicity amplitude ratios. One is for the process  $J/\psi \rightarrow b_1^- \pi^+$  and the other is for the process  $b_1^- \rightarrow \omega \pi^-$ . They are

$$\eta = \frac{B_{0,0}}{B_{1,0}}, \quad \xi = \frac{C_{0,0}}{C_{1,0}}. \tag{7}$$

where  $B_{\lambda,0}$  and  $C_{\lambda,0}$  are helicity amplitudes for the above two processes, and  $\lambda_b$  and  $\lambda_\omega$  are the helicities of the  $b_1^-$  meson and the  $\omega$  meson, respectively. Here we must note that the  $x_1-z_1$  plane and  $x_2-z_2$  plane are not the same plane.  $(\theta_f, \theta_1^*, \varphi_1^*)$  and  $(\theta_b, \varphi_b, \theta_2^*, \varphi_2^*)$  are not independent.

### 3. DOUBLE-CHANNEL COUPLING

For the process

$$e^+e^- \rightarrow J/\psi \rightarrow \begin{cases} \omega f_2 \\ b_1^- \pi \end{cases} \rightarrow \omega \pi^+ \pi^- \tag{8}$$

its  $S$  matrix element is

$$\begin{aligned} \langle \omega \pi^+ \pi^- | S - 1 | e^+ e^- \rangle &\propto \sum_{\lambda_f, \lambda_\omega, \lambda_b} \{ \langle \pi^+ \pi^- | T_3 | (f_2)_{\lambda_f} \rangle \langle \omega (f_2)_{\lambda_\omega} | T_2 | \psi_{\lambda_f} \rangle \delta(f_2) \\ &+ \langle \omega \pi^- | T_3 | (b_1^-)_{\lambda_b} \rangle \langle (b_1^-)_{\lambda_b} \pi^+ | T_2 | \psi_{\lambda_f} \rangle \delta(b_1^-) \} \cdot \langle \psi_{\lambda_f} | T_1 | e^+ e^- \rangle, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \langle \pi^+ \pi^- | T_3 | (f_2)_{\lambda_f} \rangle &= \left(\frac{2}{15}\right)^{1/2} 4g_f |p_\pi^*|^2 D_{\lambda_f, 0}^{2*}(\varphi_1^*, \theta_1^*, 0), \\ \langle (\omega)_{\lambda_\omega} \pi^- | T_3 | (b_1^-)_{\lambda_b} \rangle &= C_{\lambda_b, 0} D_{\lambda_b, \lambda_\omega}^{1*}(\varphi_2^*, \theta_2^*, -\varphi_2^*), \\ \langle (\omega)_{\lambda_\omega} (f_2)_{\lambda_f} | T_2 | \psi_{\lambda_f} \rangle &= A_{\lambda_f, \lambda_\omega} D_{\lambda_f, \lambda_f - \lambda_\omega}^{1*}(0, \theta_f, 0), \\ \langle (b_1^-)_{\lambda_b} \pi^+ | T_2 | \psi_{\lambda_f} \rangle &= B_{\lambda_b, 0} D_{\lambda_b, \lambda_b}^{1*}(\varphi_b, \theta_b - \varphi_b), \end{aligned} \tag{10}$$

$$\delta(f_2) = \frac{e^{ix_f}}{m^2 - m_f^2 + im_f\Gamma_f},$$

$$\delta(b_1) = \frac{e^{ix_b}}{m^2 - m_b^2 + im_b\Gamma_b}.$$

$|p_\pi^*|$  is the magnitude of the momentum of the final  $\pi$  meson in the  $f_2$  rest frame:

$$|p_\pi^*| = \sqrt{\left(\frac{m_f}{2}\right)^2 - m_\pi^2}. \tag{11}$$

the  $g_f$  is the coupling constant between the  $f_2$  meson and the final state ( $\pi^+\pi^-$ ). Because  $\theta_b, \varphi_b, \theta_2^*, \varphi_2^*$  and  $\theta_f, \theta_1^*, \varphi_1^*$  are not independent we must find the relation between these variables as we write down the projective angular distribution and make moment analysis.

Choosing the laboratory system (the  $J/\psi$  rest frame)  $K \equiv (e_1, e_2, e_3)$ , where  $e_3$  is the unit vector along the direction of the  $e^+$  beam. Obviously, the frame  $K_1 \equiv (x_1, y_1, z_1) = R(e_2, \theta_b)K$ , and  $y_1 \parallel e_2$ . Assuming  $e_2'$  is the moving direction of the  $\omega$  meson,  $K_1' \equiv (e_1', e_2', e_3') \equiv R(y_1, \pi)K_1$ ,  $e_2' \parallel y_1$ . In the  $K_1$  frame the direction of the  $\pi^+$  meson is described by  $(\theta_1, \varphi_1)$ . We define  $K' \equiv (x', y', z')$  and  $K_2 \equiv (x_2, y_2, z_2)$ . The  $z'$  axis is the direction of the  $e^+$  beam, the  $z_2$  axis is the moving direction of the  $b_1^-$  meson. They can be obtained from the following operations:

$$K' \equiv R(e_3, \varphi_b)K, \quad K_2 \equiv R(y', \theta_b)K'. \tag{12}$$

Here  $R$  is the rotation operator. For example,  $R(y', \theta_b)$  represent a rotation through an angle  $\theta_b$  in the positive direction around the  $y'$  axis. So  $e_3 \parallel z'$  is the direction of the  $e^+$  beam, the  $x' - z'$  plane and  $x_2 - z_2$  plane are the same plane. Because the moving direction of the  $\omega$  meson is described by  $(\theta_2, \varphi_2)$  in the  $K_2$  frame, we have

$$K_1' = R(\varphi_2, \theta_2, \gamma_2)K_2. \tag{13}$$

We can set up the  $K_1''$  frame from the following operation:

$$K_1'' = R(\varphi_1, \theta_1, \gamma_1)K_1 \equiv (e_1'', e_2'', e_3''). \tag{14}$$

where the  $e_3''$  axis is the moving direction of the  $\pi^+$  meson. We can put the  $e^+$  beam in the  $e_1'' - e_3''$  plane through selecting the Euler angle  $\gamma_1$ . Then there is the following relation between the  $K_2$  frame and the  $K_1''$  frame:

$$K_2 \equiv R(e_2'', \pi)K_1''. \tag{15}$$

From Eqs.(12)-(15) we can get the relations:

$$\begin{aligned} \cos\theta_b &= \sin\theta_f \cos\varphi_1 \sin\theta_1 - \cos\theta_f \cos\theta_1, \\ \sin\theta_b &= +\sqrt{1 - \cos^2\theta_b}, \\ \sin\gamma_1 &= \sin\theta_f \sin\varphi_1 / \sin\theta_b, \\ \cos\gamma_1 &= -\frac{\sin\theta_f \cos\theta_1 \cos\varphi_1 + \cos\theta_f \sin\theta_1}{\sin\theta_b}, \\ \sin\varphi_b &= -\sin\theta_1 \sin\varphi_1 / \sin\theta_b, \\ \cos\varphi_b &= -(\sin\theta_f \cos\theta_1 + \cos\theta_f \sin\theta_1 \cos\varphi_1) / \sin\theta_b, \end{aligned} \tag{16}$$

and

$$\begin{aligned}\theta_2 &= \theta_1, \\ \cos\gamma_2 &= -\cos\varphi_1, \quad \sin\gamma_2 = -\sin\varphi_1, \\ \cos\varphi_2 &= -\cos\gamma_1, \quad \sin\varphi_2 = -\sin\gamma_1.\end{aligned}\tag{17}$$

where the Euler angles  $\theta_1, \theta_2, \theta_f, \theta_b$  take  $0 \rightarrow \pi$ , and the other Euler angles take  $0 \rightarrow 2\pi$ . The relations between  $\theta_1^*, \varphi_1^*, \theta_2^*, \varphi_2^*$  in the Eq.(10) and  $\theta_1, \varphi_1, \theta_2, \varphi_2$  are

$$\begin{aligned}\cos\theta_1^* &= \gamma_f (|p_\pi| \cos\theta_1 - \beta_f E_\pi) / |p_\pi^*|, \\ \sin\theta_1^* &= |p_\pi| \sin\theta_1 / |p_\pi^*|, \\ \varphi_1^* &= \varphi_1, \\ \cos\theta_2^* &= \gamma_b (|p_\omega| \cos\theta_2 - \beta_b E_\omega) / |p_\omega^*|, \\ \sin\theta_2^* &= |p_\omega| \sin\theta_2 / |p_\omega^*|, \\ \varphi_2^* &= \varphi_2,\end{aligned}\tag{18}$$

where

$$\begin{aligned}\beta_f &= \frac{|p_f|}{E_f}, \quad \gamma_f = \frac{E_f}{m_f}, \\ |p_f| &= [m_j^2 - (m_f + m_\omega)^2]^{1/2} [m_j^2 - (m_f - m_\omega)^2]^{1/2} / 2m_j = |p_\omega|, \\ E_f &= (m_j^2 + m_f^2 - m_\omega^2) / 2m_j, \quad E_\omega = (m_j^2 + m_\omega^2 - m_f^2) / 2m_j, \\ \beta_b &= \frac{|p_b|}{E_b}, \quad \gamma_b = \frac{E_b}{m_b}, \\ |p_b| &= [m_j^2 - (m_b + m_\pi)^2]^{1/2} [m_j^2 - (m_b - m_\pi)^2]^{1/2} / 2m_j = |p_\pi|, \\ E_b &= (m_j^2 + m_b^2 - m_\pi^2) / 2m_j, \quad E_\pi = (m_j^2 + m_\pi^2 - m_b^2) / 2m_j.\end{aligned}\tag{19}$$

And the  $|p_\pi|$  and  $E_\pi$  are the magnitude of the momentum and the energy of the  $\pi^+$  meson in the  $J/\psi$  rest frame, respectively. The  $|p_\omega|$  and  $E_\omega$  are the magnitude of the momentum and the energy of the  $\omega$  meson in the  $J/\psi$  rest frame. The  $|p_\omega^*|$  is the magnitude of the momentum of the final  $\omega$  meson in the  $b_1$  rest frame

$$|p_\omega^*| = [m_b^2 - (m_\omega + m_\pi)^2]^{1/2} [m_b^2 - (m_\omega - m_\pi)^2]^{1/2} / 2m_b.\tag{20}$$

The helicity formalism of the angular distribution for the process (8) can be obtained from Eqs.(9) and (10). After considering the double-channel coupling the formalism is as follows:

$$\begin{aligned}W(\theta_f, \theta_b, \varphi_b, \theta_1^*, \varphi_1^*, \theta_2^*, \varphi_2^*) &\propto \sum_{\substack{\lambda_f, \lambda_f', \lambda_b \\ \lambda_f, \lambda_b', \lambda_c}} \delta_{\lambda_f, \pm 1} \left\{ \frac{5}{4\pi} \cdot \alpha \cdot A_{\lambda_f, \lambda_c} A_{\lambda_f', \lambda_c}^* D_{\lambda_f, \lambda_f - \lambda_c}^{1*}(0, \theta_f, 0) \right. \\ &\cdot D_{\lambda_f, \lambda_f - \lambda_c}^1(0, \theta_f, 0) \cdot D_{\lambda_f, 0}^{2*}(\varphi_1^*, \theta_1^*, 0) \cdot D_{\lambda_f', 0}^2(\varphi_1^*, \theta_1^*, 0) \\ &+ \frac{3}{4\pi} \cdot \beta (B_{\lambda_b, 0} C_{\lambda_b, 0}) (B_{\lambda_b', 0} C_{\lambda_b', 0})^* D_{\lambda_b, \lambda_b}^{1*}(\varphi_b, \theta_b, -\varphi_b) D_{\lambda_b, \lambda_b'}^1(\varphi_b, \theta_b, -\varphi_b) \\ &\left. \cdot D_{\lambda_b, \lambda_b}^{1*}(\varphi_2^*, \theta_2^*, -\varphi_2^*) D_{\lambda_b', \lambda_b}^1(\varphi_2^*, \theta_2^*, -\varphi_2^*) \right\}\end{aligned}$$

$$\begin{aligned}
& + \frac{\sqrt{15}}{2\pi} \operatorname{Re} \left[ \gamma \cdot A_{\lambda_f, \lambda_b} (B_{\lambda_b, 0} C_{\lambda_b, 0})^* D_{\lambda_f, \lambda_f - \lambda_b}^{1*} (0, \theta_f, 0) D_{\lambda_f, \lambda_b}^1 (\varphi_b, \theta_b, -\varphi_b) \right. \\
& \cdot D_{\lambda_f, 0}^{2*} (\varphi_1^*, \theta_1^*, 0) D_{\lambda_b, \lambda_b}^1 (\varphi_2^*, \theta_2^*, -\varphi_2^*) \left. \right] \\
& = \frac{5}{4\pi} \cdot \alpha \cdot W_2 (\theta_f, \theta_1^*, \varphi_1^*) + \frac{3}{4\pi} \cdot \beta \cdot W_1 (\theta_b, \varphi_b, \theta_2^*, \varphi_2^*) \\
& + \frac{\sqrt{15}}{2\pi} \cdot \operatorname{Re} (\gamma) \cdot W_{\text{D.C}} (\theta_f, \theta_b, \varphi_b, \theta_1^*, \varphi_1^*, \theta_2^*, \varphi_2^*).
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\alpha &= \frac{32}{15} |p_f^*|^4 \cdot g_f^2 \cdot |\delta(f_2)|^2, \\
\beta &= |\delta(b_1)|^2, \\
\gamma &= 4 \cdot \left( \frac{2}{15} \right)^{1/2} \cdot |p_f^*|^2 \cdot g_f [\delta(f_2) \delta^*(b_1)].
\end{aligned} \tag{22}$$

and the  $W_2 (\theta_f, \theta_1^*, \varphi_1^*)$  and the  $W_1 (\theta_b, \varphi_b, \theta_2^*, \varphi_2^*)$  are given by Eqs.(3) and (6), respectively. The part of the double-channel coupling is

$$\begin{aligned}
W_{\text{D.C}} (\theta_f, \theta_b, \varphi_b, \theta_1^*, \varphi_1^*, \theta_2^*, \varphi_2^*) &\propto \sum_{\lambda_f, \lambda_f, \lambda_b, \lambda_b} \delta_{\lambda_f, \pm 1} A_{\lambda_f, \lambda_b} (B_{\lambda_b, 0} C_{\lambda_b, 0}) \\
&\times d_{\lambda_f, \lambda_f - \lambda_b}^1 (\theta_f) d_{\lambda_f, 0}^2 (\theta_1^*) d_{\lambda_f, \lambda_b}^1 (\theta_b) d_{\lambda_b, \lambda_b}^1 (\theta_2^*) \\
&\times \cos [\lambda_f \varphi_1^* - (\lambda_f - \lambda_b) \varphi_b - (\lambda_b - \lambda_b) \varphi_2^*].
\end{aligned} \tag{23}$$

The parity conservation and the time-reversal invariance are used in obtaining above equations. Each group of helicity amplitudes  $A_{\lambda_f, \lambda_b}$  and  $(B_{\lambda_b, 0} C_{\lambda_b, 0})$  are relatively real [7]. Their phase factors are included in  $\delta(f_2)$  and  $\delta(b_1)$ , respectively. Then independent variables in the Eq.(21) include nine independent helicity amplitudes  $A_{2,1}, A_{1,1}, A_{0,1}, A_{1,0}, A_{0,0}, B_{1,0}, B_{0,0}, C_{1,0}$ , and  $C_{0,0}$  and a phase difference  $(\chi_f - \chi_b)$ .

Using the relations given by Eqs.(16-18), and selecting independent variables  $(\theta_f, \theta_1^*, \varphi_1^*)$ , we can obtain the projective angular distributions  $W(\theta_f)$ ,  $W(\theta_1^*)$ ,  $W(\varphi_1^*)$  and perform the moment analysis.

#### 4. DISCUSSION

As an example, we discuss the multichannel coupling problem for the process  $J / \psi \rightarrow \omega \pi^+ \pi^-$ . BES collaboration can make further analysis on the basis of the original work [5] and obtain a more precise result.

For the interesting glueball candidate  $\iota / \eta$  (1440), the multistate structure discovered in the following three-step two-body decay process [8]

$$e^+e^- \rightarrow J / \psi \rightarrow \gamma + X, X \rightarrow K^* \bar{K}, K^* \rightarrow K\pi,$$

or

$$e^+e^- \rightarrow J / \psi \rightarrow \gamma + X, X \rightarrow a_0 \pi, a_0 \rightarrow K\bar{K}. \tag{24}$$

causes much attention. In order to precisely measure the multistate structure we must study the

multichannel coupling except we consider directly the three-body decay [9]. If the final state is  $K^+K^-\pi^0$  there will exist  $K^{*\pm}K^\mp$ ,  $K^{*\pm} \rightarrow K^\pm\pi^0$ , and  $a_0\pi^0$ ,  $a_0 \rightarrow K^+K^-$  three-channel coupling. Because  $X$ , the  $J/\psi$  radiative decay products, represent many resonances the discussion of the multichannel coupling in the multistate structure is more complicated. We will study it further in future papers.

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