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以子午面上磁场表示的空间磁场分布

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摘 要

本文推导了在柱坐标系下用 $\phi=\phi_0$ 子午面上的磁场来表示的空间场(平面对称与非对称)的普遍泰勒级数展开式,平面场可由解析或离散形式给出。

一、引 言

在带电粒子光学中,要研究光学系统的光学特性,首先要知道系统的空间磁场分布.在旁轴光学中,空间场常常用参考轨迹上的场分布及其各阶法向导数求出[1]. 对于非旁轴光学系统,如: 255°β 谱仪[2],环形磁 β 谱仪[3,4]等,用上述方法求空间场,在某些区域就很难满足要求,因而提出了用某一平面上的场分布来表示空间的场分布.在直角坐标系下用中间平面上的场表示的空间场已由 L. O. Love 等人[5]导出. 文献[6]和[7]则给出了柱坐标系下由 $z=z_0$ 平面上的场表示的空间场的泰勒级数展开式,场对该平面可以是对称的也可以是非对称的,[6]中还介绍了相应的计算机程序.这些结果适用于 $z=z_0$ 平面附近带电粒子光学特性的研究.但在某些情况下,光学系统中粒子的主入射平面(包含主轨迹的平面)在柱坐标系中是 $\phi=$ 常数的平面,如环形磁 β 谱仪等.为此,本文在柱坐标系中推导了用 $\phi=\phi_0$ 平面上的场表示的空间场的普遍泰勒级数展开式, $\phi=\phi_0$ 平面上的场可由解析式给出,也可由离散点上的场值给出.对以离散点给出的场分布,本文采用了双三次样条[8]拟合.并在 Cyber-170/825 机上编写了计算空间场的计算机程序.

二、空间场泰勒级数展开式

1. 展开式及系数表达式

已知在柱坐标系 (r,ϕ,z) 中 $\phi=0$ 平面上的场分布

$$\boldsymbol{B}=B_r(r,0,z)\boldsymbol{e}_r+B_\phi(r,0,z)\boldsymbol{e}_\phi+B_z(r,0,z)\boldsymbol{e}_z,$$

当 $\phi\neq0$ 时空间场的三个分量可由下列泰勒级数展开式表示:

$$B_r(r,\phi,z)=\sum_{n=0}^{\infty}\left[\frac{1}{n!}\frac{\partial^n B_r(r,\phi,z)}{\partial\phi^n}\bigg|_{\phi=0}\right]\phi^n=\sum_{n=0}^{\infty}a_{r,n}(r,z)\phi^n, \tag{1}$$

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$$B_\phi(r,\phi,z)=\sum_{n=0}^{\infty}\left[\frac{1}{n!}\frac{\partial^n B_\phi(r,\phi,z)}{\partial\phi^n}\bigg|_{\phi=0}\right]\phi^n=\sum_{n=0}^{\infty}a_{\phi,n}(r,z)\phi^n, \tag{2}$$

$$B_z(r,\phi,z)=\sum_{n=0}^{\infty}\left[\frac{1}{n!}\frac{\partial^n B_z(r,\phi,z)}{\partial\phi^n}\bigg|_{\phi=0}\right]\phi^n=\sum_{n=0}^{\infty}a_{z,n}(r,z)\phi^n. \tag{3}$$

为书写方便，系数 $a_{r,n}(r,z)$，$a_{\phi,n}(r,z)$ 和 $a_{z,n}(r,z)$ 将分别简写为 $a_{r,n}$，$a_{\phi,n}$ 和 $a_{z,n}$。

设所考虑的空间为无源空间，则对于静磁场由 Maxwell 方程有下列关系式：

$$\nabla\times\boldsymbol{B}=0, \tag{4}$$

$$\nabla\cdot\boldsymbol{B}=0. \tag{5}$$

由此可得 $\nabla^2\boldsymbol{B}=0$，即：

$$\left(\nabla^2 B_r-\frac{1}{r^2}B_r-\frac{2}{r^2}\frac{\partial B_\phi}{\partial\phi}\right)\boldsymbol{e}_r+\left(\nabla^2 B_\phi-\frac{1}{r^2}B_\phi+\frac{1}{r^2}\frac{\partial B_r}{\partial\phi}\right)\boldsymbol{e}_\phi$$
$$+\nabla^2 B_z\boldsymbol{e}_z=0$$

所以

$$\nabla^2 B_\phi-\frac{1}{r^2}B_\phi+\frac{2}{r^2}\frac{\partial B_r}{\partial\phi}=0. \tag{6}$$

将(1)式和(2)式代入(6)式并利用柱坐标系下对标量的拉氏算符：

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}$$

可得：

$$r^2\left(\sum_{n=2}^{\infty}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}a_{\phi,n-2}\phi^{n-2}+\frac{1}{r^2}\sum_{n=2}^{\infty}n(n-1)a_{\phi,n}\phi^{n-2}\right.$$
$$\left.+\sum_{n=2}^{\infty}\frac{\partial^2}{\partial z^2}a_{\phi,n-2}\phi^{n-2}\right)-\sum_{n=2}^{\infty}a_{\phi,n-2}\phi^{n-2}+2\sum_{n=2}^{\infty}(n-1)a_{r,n-1}\phi^{n-2}=0,$$

因而

$$\left.\begin{array}{l}\left(r\dfrac{\partial}{\partial r}+r^2\dfrac{\partial z}{\partial r^2}+r^2\dfrac{\partial^2}{\partial z^2}-1\right)a_{\phi,n-2}+n(n-1)a_{\phi,n}\\ \qquad+2(n-1)a_{r,n-1}=0\\ \qquad\qquad(n=2,3,4,\cdots).\end{array}\right\} \tag{7}$$

由(4)式可得 $a_{r,n}$，$a_{\phi,n}$，$a_{z,n}$ 的关系式：

$$a_{r,n}=\frac{1}{n}\left(a_{\phi,n-1}+r\frac{\partial}{\partial r}a_{\phi,n-1}\right),\quad(n=1,2,3,\cdots) \tag{8}$$

$$a_{z,n}=\frac{r}{n}\frac{\partial}{\partial z}a_{\phi,n-1},\quad(n=1,2,3,\cdots). \tag{9}$$

将(8)式代入(7)式即得 $a_{\phi,n}$ 的递推公式：

$$\left.\begin{array}{l}a_{\phi,n}=-\dfrac{1}{n(n-1)}\left(1+3r\dfrac{\partial}{\partial r}+r^2\dfrac{\partial^2}{\partial r^2}+r^2\dfrac{\partial^2}{\partial z^2}\right)a_{\phi,n-2},\\ \qquad\qquad(n=2,3,4,\cdots)\end{array}\right\} \tag{10}$$

当 $n=0$ 时，由(2)式：

$$a_{\phi,0}=B_\phi(r,0,z)$$

代人(10)式可得 $B_\phi(r,\phi,z)$ 展开式偶次幂项系数 $a_{\phi,2n}$ 的表达式:

$$\left.\begin{aligned} a_{\phi,2n} &= \frac{(-1)^n}{(2n)!}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n B_\phi(r,0,z), \\ &(n=0,1,2,\cdots) \end{aligned}\right\} \quad (11)$$

当 $n=1$ 时,由(2)和(5)式:

$$\begin{aligned} a_{\phi,1} &= \frac{\partial}{\partial\phi}B_\phi(r,\phi,z)|_{\phi=0} \\ &= -\left[\left(1+r\frac{\partial}{\partial r}\right)B_r(r,0,z)+r\frac{\partial}{\partial z}B_z(r,0,z)\right], \end{aligned}$$

代人(10)式可得奇次幂项系数 $a_{\phi,2n+1}$ 的表达式:

$$\begin{aligned} a_{\phi,2n+1} &= \frac{(-1)^{n+1}}{(2n+1)!}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n \\ &\times\left[\left(1+r\frac{\partial}{\partial r}\right)B_r(r,0,z)+r\frac{\partial}{\partial z}B_z(r,0,z)\right]. \\ &(n=0,1,2,\cdots) \end{aligned} \quad (12)$$

系数 $a_{r,n}$ 和 $a_{z,n}$ 的表达式利用 (1),(3) 以及(8),(9),(11)和(12)式不难求得:

$$\left.\begin{aligned} a_{r,0} &= B_r(r,0,z), \\ a_{r,2n} &= \frac{(-1)^n}{(2n)!}\left(1+r\frac{\partial}{\partial r}\right)\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^{n-1} \\ &\times\left[\left(1+r\frac{\partial}{\partial r}\right)B_r(r,0,z)+r\frac{\partial}{\partial z}B_z(r,0,z)\right] \\ &(n=1,2,3,\cdots) \end{aligned}\right\} \quad (13)$$

$$\left.\begin{aligned} a_{r,2n+1} &= \frac{(-1)^n}{(2n+1)!}\left(1+r\frac{\partial}{\partial r}\right)\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n B_\phi(r,0,z), \\ &(n=0,1,2,\cdots) \end{aligned}\right\} \quad (14)$$

$$\left.\begin{aligned} a_{z,0} &= B_z(r,0,z), \\ a_{z,2n} &= \frac{(-1)^n r}{(2n)!}\frac{\partial}{\partial z}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^{n-1} \\ &\times\left[\left(1+r\frac{\partial}{\partial r}\right)B_r(r,0,z)+r\frac{\partial}{\partial z}B_z(r,0,z)\right], \\ &(n=1,2,3,\cdots) \end{aligned}\right\} \quad (15)$$

$$\left.\begin{aligned} a_{z,2n+1} &= \frac{(-1)^n r}{(2n+1)!}\frac{\partial}{\partial z}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n B_\phi(r,0,z), \\ &(n=0,1,2,\cdots) \end{aligned}\right\} \quad (16)$$

将 (11)—(16) 式代人 (1)—(3) 式即得以 $\phi=0$ 平面上 $B_r(r,0,z)$, $B_\phi(r,0,z)$, $B_z(r,0,z)$ 及其对 r, z 偏导数表示的空间场分布。

2. $\phi=0$ 平面为对称平面时的特殊情况

当 $\phi=0$ 平面为对称平面时, $B_r(r,0,z)\equiv B_z(r,0,z)\equiv 0$,场展开式系数表达式

(11)—(16)退化成如下形式：

$$
\left.\begin{aligned}
&a_{\phi,2n}=\frac{(-1)^n}{(2n)!}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n B_\phi(r,0,z),\\
&\qquad (n=0,1,2,\cdots)
\end{aligned}\right\}\tag{11'}
$$

$$
a_{\phi,2n+1}\equiv 0,\quad (n=0,1,2,\cdots)\tag{12'}
$$

$$
a_{r,2n}\equiv 0,\quad (n=0,1,2,\cdots)\tag{13'}
$$

$$
\left.\begin{aligned}
&a_{r,2n+1}=\frac{(-1)^n}{(2n+1)!}\left(1+r\frac{\partial}{\partial r}\right)\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n B_\phi(r,0,z),\\
&\qquad (n=0,1,2,\cdots)
\end{aligned}\right\}\tag{14'}
$$

$$
a_{z,2n}\equiv 0,\quad (n=0,1,2,\cdots)\tag{15'}
$$

$$
\left.\begin{aligned}
&a_{z,2n+1}=\frac{(-1)^n r}{(2n+1)!}\frac{\partial}{\partial z}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)^n B_\phi(r,0,z),\\
&\qquad (n=0,1,2,\cdots)
\end{aligned}\right\}\tag{16'}
$$

磁场展开式为：

$$
\left.\begin{aligned}
B_r(r,\phi,z)&=\sum_{n=0}^{\infty}a_{r,2n+1}\phi^{2n+1},\\
B_\phi(r,\phi,z)&=\sum_{n=0}^{\infty}a_{\phi,2n}\phi^{2n},\\
B_z(r,\phi,z)&=\sum_{n=0}^{\infty}a_{z,2n+1}\phi^{2n+1}.
\end{aligned}\right\}\tag{17}
$$

3. 前六项场系数的具体表达式

由式 (11)—(16) 不难导出级数展开式各项系数的具体表达式。其前六项系数分别为：

0 次项：

$$a_{r,0}=B_r(r,0,z)$$

$$a_{\phi,0}=B_\phi(r,0,z)$$

$$a_{z,0}=B_z(r,0,z)$$

1 次项：

$$a_{r,1}=B_\phi(r,0,z)+r\frac{\partial}{\partial r}B_\phi(r,0,z)$$

$$a_{\phi,1}=-\left[\left(1+\frac{\partial}{\partial r}\right)B_r(r,0,z)+r\frac{\partial}{\partial z}B_z(r,0,z)\right]$$

$$a_{z,1}=r\frac{\partial}{\partial z}B_\phi(r,0,z)$$

2 次项：

$$a_{r,2}=\frac{1}{2}a_{\phi,1}-\frac{r}{2}\left[\left(2\frac{\partial}{\partial r}+r\frac{\partial^2}{\partial r^2}\right)B_r(r,0,z)\right.$$

$$+\left(\frac{\partial}{\partial z}+r\frac{\partial^2}{\partial r\partial z}\right)B_z(r,0,z)\Big]$$

$$a_{\phi,2}=-\frac{1}{2}\left(1+3r\frac{\partial}{\partial r}+r^2\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^2}{\partial z^2}\right)B_\phi(r,0,z)$$

$$a_{z,2}=-\frac{r}{2}\left[\left(\frac{\partial}{\partial z}+r\frac{\partial^2}{\partial r\partial z}\right)B_r(r,0,z)+r\frac{\partial^2}{\partial z^2}B_z(r,0,z)\right]$$

3 次项:

$$a_{r,3}=\frac{1}{3}a_{\phi,2}-\frac{r}{3!}\Big[4\frac{\partial}{\partial r}+5r\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^3}{\partial r^3}+2r\frac{\partial^2}{\partial z^2}$$
$$+r^2\frac{\partial^3}{\partial r\partial z^2}\Big]B_\phi(r,0,z)$$

$$a_{\phi,3}=\frac{1}{3!}\Big[\left(1+7r\frac{\partial}{\partial r}+6r^2\frac{\partial^2}{\partial r^2}+r^3\frac{\partial^3}{\partial r^3}+r^2\frac{\partial^2}{\partial z^2}+r^3\frac{\partial^3}{\partial r\partial z^2}\right)B_r(r,0,z)$$
$$+\left(4r\frac{\partial}{\partial z}+5r^2\frac{\partial^2}{\partial r\partial z}+r^3\frac{\partial^3}{\partial r^2\partial z}+r^3\frac{\partial^3}{\partial z^3}\right)B_z(r,0,z)\Big]$$

$$a_{z,3}=\frac{1}{3}a_{\phi,2}-\frac{r}{3!}\Big[4\frac{\partial}{\partial r}+5r\frac{\partial^2}{\partial r^2}+r^2\frac{\partial^3}{\partial r^3}+2r\frac{\partial^2}{\partial z^2}$$
$$+r^2\frac{\partial^3}{\partial r\partial z^2}\Big]B_\phi(r,0,z)$$

4 次项:

$$a_{r,4}=\frac{1}{4}a_{\phi,3}+\frac{r}{4!}\Big[\Big(8\frac{\partial}{\partial r}+19r\frac{\partial^2}{\partial r^2}+9r^2\frac{\partial^3}{\partial r^3}+r^3\frac{\partial^4}{\partial r^4}+2r\frac{\partial^2}{\partial z^2}$$
$$+4r^2\frac{\partial^3}{\partial r\partial z^2}+r^3\frac{\partial^4}{\partial r^2\partial z^2}\Big)B_r(r,0,z)+\Big(4\frac{\partial}{\partial z}+14r\frac{\partial^2}{\partial r\partial z}$$
$$+8r^2\frac{\partial^3}{\partial r^2\partial z}+r^3\frac{\partial^4}{\partial r^3\partial z}+3r^2\frac{\partial^3}{\partial z^3}+r^3\frac{\partial^4}{\partial r\partial z^3}\Big)B_z(r,0,z)\Big]$$

$$a_{\phi,4}=\frac{1}{4!}\Big(1+15r\frac{\partial}{\partial r}+25r^2\frac{\partial^2}{\partial r^2}+10r^3\frac{\partial^3}{\partial r^3}+r^4\frac{\partial^4}{\partial r^4}+10r^2\frac{\partial^2}{\partial z^2}$$
$$+10r^3\frac{\partial^3}{\partial r\partial z^2}+2r^4\frac{\partial^4}{\partial r^2\partial z^2}+r^4\frac{\partial^4}{\partial z^4}\Big)B_\phi(r,0,z)$$

$$a_{z,4}=\frac{1}{4!}\Big[\Big(\frac{\partial}{\partial z}+7r\frac{\partial^2}{\partial r\partial z}+6r^2\frac{\partial^3}{\partial r^2\partial z}+r^3\frac{\partial^4}{\partial r^3\partial z}+r^2\frac{\partial^3}{\partial z^3}$$
$$+r^3\frac{\partial^4}{\partial r\partial z^3}\Big)B_r(r,0,z)+\Big(4r\frac{\partial^2}{\partial z^2}+5r^2\frac{\partial^3}{\partial r\partial z^2}+r^3\frac{\partial^4}{\partial r^2\partial z^2}$$
$$+r^3\frac{\partial^4}{\partial z^4}\Big)B_z(r,0,z)\Big]$$

5 次项:

$$a_{r,5}=\frac{1}{5}a_{\phi,4}+\frac{r}{5!}\Big(16\frac{\partial}{\partial r}+65r\frac{\partial^2}{\partial r^2}+55r^2\frac{\partial^3}{\partial r^3}+14r^3\frac{\partial^4}{\partial r^4}+r^4\frac{\partial^5}{\partial r^5}$$
$$+20r\frac{\partial^2}{\partial z^2}+40r^2\frac{\partial^3}{\partial r\partial z^2}+18r^3\frac{\partial^4}{\partial r^2\partial z^2}+2r^4\frac{\partial^5}{\partial r^3\partial z^2}$$

$$+ 4r^3 \frac{\partial^4}{\partial z^4} + r^4 \frac{\partial^5}{\partial r \partial z^4}\Big) B_\phi(r, 0, z)$$

$$a_{\phi,5} = -\frac{1}{20} a_{\phi,3} - \frac{1}{5!}\Big[\Big(24r \frac{\partial}{\partial r} + 84r^2 \frac{\partial^2}{\partial r^2} + 64r^3 \frac{\partial^3}{\partial r^3} + 15r^4 \frac{\partial^4}{\partial r^4}$$

$$+ r^5 \frac{\partial^5}{\partial r^5} + ar^2 \frac{\partial^2}{\partial z^2} + 29r^3 \frac{\partial^3}{\partial r \partial z^2} + 16r^4 \frac{\partial^4}{\partial r^2 \partial z^2}$$

$$+ 2r^5 \frac{\partial^5}{\partial r^3 \partial z^2} + r^4 \frac{\partial^4}{\partial z^4} + r^5 \frac{\partial^5}{\partial r \partial z^4}\Big) B_r(r, 0, z)$$

$$+ \Big(12r \frac{\partial}{\partial z} + 60r^2 \frac{\partial^2}{\partial r \partial z} + 54r^3 \frac{\partial^3}{\partial r^2 \partial z} + 14r^4 \frac{\partial^4}{\partial r^3 \partial z} + r^5 \frac{\partial^5}{\partial r^4 \partial z}$$

$$+ 19r^3 \frac{\partial^3}{\partial z^3} + 14r^4 \frac{\partial^4}{\partial r \partial z^3} + 2r^5 \frac{\partial^5}{\partial r^2 \partial z^3} + r^5 \frac{\partial^5}{\partial z^5}\Big) B_z(r, 0, z)\Big]$$

$$a_{z,5} = \frac{r}{5!}\Big(\frac{\partial}{\partial z} + 15r \frac{\partial^2}{\partial r \partial z} + 25r^2 \frac{\partial^3}{\partial r^2 \partial z} + 10r^3 \frac{\partial^4}{\partial r^3 \partial z} + r^4 \frac{\partial^5}{\partial r^4 \partial z}$$

$$+ 10r^2 \frac{\partial^3}{\partial z^3} + 10r^3 \frac{\partial^4}{\partial r \partial z^3} + 2r^4 \frac{\partial^5}{\partial r^2 \partial z^3} + r^4 \frac{\partial^5}{\partial z^5}\Big) B_\phi(r, 0, z)$$

三、场的双三次样条函数拟合

在以平面场表示的空间场泰勒级数展开式中，平面场可由解析形式给出，也可由离散形式给出. 对于前者，可直接把场的解析式代入展开式，求得空间场分布. 对于后者，我们首先采用双三次样条函数拟合平面场，然后再代入级数展开式.

设给定 $\phi = 0$ 平面上离散点 (r_i, z_j)($r_1 < r_2 < \cdots < r_M, z_1 < z_2 < \cdots < z_N$)上的磁感应强度 B_r, B_ϕ, B_z，则在该平面上任一子域 $r_i < r \leqslant r_{i+1}, (i = 1, 2, \cdots, M), z_j < z \leqslant z_{j+1}$，$(j = 1, 2, \cdots N)$ 内的场分布可由双三次样条插值

$$B(r, 0, z) = \sum_{n=0}^{3} \sum_{m=0}^{3} A_{i,j,m,n}(r - r_i)^m (z - z_j)^n \tag{18}$$

求得. 其中 $A_{i,j,m,n}$ 为样条系数，它可根据插值函数在节点上偏导数的连续性和场的一、二类边界条件确定. 场的二类边界条件(即场对 r 或 z 的一阶导数)本文采用四点三次插值求出. 双三次样条插值保证了在整个插值区或内插值函数对 r，z 的偏导数到二阶，混合偏导数到四阶连续.

将 (18) 式对 r，z 分别求 ν 和 s 次偏导数：

$$\frac{\partial^{\nu+s}}{\partial r^\nu \partial z^s} B(r, 0, z) = \sum_{m=\nu}^{3} \sum_{n=s}^{3} m \cdot n \cdots (m - \nu + 1)$$

$$\times (n - s + 1)(r - r_i)^{m-\nu} \cdot (z - z_j)^{n-s}$$

代入(11)—(16)式以及(1)—(3)式就可以求得空间场分布.

在 Cyber-170/825 机上编写了平面场以离散形式给出的空间场计算程序 SPMAG，并对当场展开式取有限项及采用双三次样条拟合平面场时空间场计算的准确度进行了验算.

1）场展开式取有限项时准确度的验算

验算采用 $\frac{1}{r}$ 场，$B_\phi(r, \phi, z) = B_0 \frac{r_0}{r}$，$B_r(r,\phi,z) = B_z(r,\phi,z) = 0$，其中，$B_0 = 1G$，$r_0 = 1$ cm．方法是：将 $\phi=0$ 平面上场 $B_\phi(r, 0, z) = B_0 \frac{r_0}{r}$，$B_r(r,0,z) = 0$，$B_z(r, 0, z) = 0$ 代入空间场展开式取到 ϕ 的五次项求得空间场，然后与 $\frac{1}{r}$ 场比较．结果表明：当 $\phi \leqslant 5°$ 时相对误差小于 6.0×10^{-5}，当 $\phi \leqslant 3°$ 时相对误差小于 7.0×10^{-6}．

2）双三次样条函数拟合准确度的验算

用 $\phi = 0$ 对称平面上场的两种不同分布：

$$B_\phi(r, z) = B_0 \frac{r_0}{r}, \tag{19}$$

$$B_\phi(r, z) = B_0 \left[\frac{z}{r} + \cos\left(\frac{z}{L_0}\pi\right)\right] \tag{20}$$

对样条拟合准确度进行了验算，其中，$B_0 = 1G$，$r_0 = 1$ cm，$L_0 = 90$ cm．首先用上述两种场分布给 $\phi = 0$ 平面上节点 (r_i, z_i) $(r_1 = 7\text{cm} < r_2 < \cdots < r_m = 31$ cm，$z_1 = 1$ cm$< z_2 < \cdots < z_N = 90$ cm，$\Delta r = 1$ cm，$\Delta z = 3$ cm$)$ 上的场赋值，用样条拟合平面场并代入空间场展开式中求出空间场，然后再和解析分布计算出的空间场进行比较．

对于 $1/r$ 场，用样条拟合求得空间场分布与 $1/r$ 场比较表明：当 $\phi \leqslant 5°$ 时，相对误差小于 1.52×10^{-4}；当 $\phi \leqslant 3°$ 时，相对误差小于 3.87×10^{-5}．

对于式(20)给出的平面场分布，样条函数拟合结果与将(20)式代入空间场展开式取到五次项计算结果相比，当 $\phi \leqslant 3°$ 时相对偏差不大于 3.0×10^{-4}．

四、结束语

本文给出了在柱坐标系中用 $\phi = 0$ 平面上磁场表示的普遍泰勒级数展开式．场对该平面可以对称也可以非对称，只要给出平面上的场分布（解析形式或离散形式），就可以求出空间场分布．当级数取有限项及采用双三次样条函数拟合离散场值时，空间场计算具有相当高的精度．与利用数值法直接计算空间场相比，计算量要小得多．展开式特别适用于带电粒子光学的研究．

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SPATIAL DISTRIBUTION OF MAGNETIC FIELD EXPRESSED BY THE FIELD ON MERIDIANAL PLANE

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ABSTRACT

A general expansion in Taylor series of space magnetic field (median plane symetry or nonsymetry) expressed in terms of the field in the $\phi=\phi_0$ meridian plane, which may be expressed analytically or by field values at discrete nodes, is derived in cylindrical coordinate system.